Wavelets-based methods for numerical solution of variational inequalities arising in elastoplasticity

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Abstract

Wavelet transform and wavelet bases [11, 16] were originally conceived as a powerful tool for signal and image processing. More recently, wavelet analysis has been applied to the numerical solution of partial differential equations arising in various areas of engineering and physics. In particular, with wavelets based methods one can realize an effective multiscale analysis of functions and operators [6]. Within this framework it is possible to build suitable preconditioners for a given problem as well as consider new fast and higher order adaptive algorithms by using wavelet bases [9].

This kind of methods has been studied both from the theoretical and the computational point of view. As to applications to steady state problems, a wavelet-Galerkin approach was considered in [5], collocation-wavelet methods were introduced in [2], the boundary element framework was studied in [20], preconditioning in [7], and fast operator compression in [10]. For time dependent problems, noteworthy contributions include [1, 3, 8, 4, 15, 17].

This work follows other papers regarding the application of wavelet analysis to differential systems arising in structural mechanics [18, 19]. In particular, we will consider the numerical approximation of elastoplastic problems. We will adopt a mixed formulation and analyze the associated mixed variational inequality. There is now a large literature on the numerical approximation of variational inequalities considered by different authors using "conventional numerical methods", see for example the works [12, 14].

The aim of this contribution is to consider adaptive wavelet discretizations of mixed variational problems in form of variational inequalities [13].

By considering a two-field functional wherein displacements and plastic strains are independently approximated by wavelets with compact support, we can establish adaptive algorithms that are well suited to capture the localized behavior of the plastic flow.

One-dimensional truss structures as well as plane–stress two-dimensional test cases are numerically studied. Finally, some possible developments are sketched.

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