Mathematical modeling of industrial processes on the basis of unknown boundary problems: parametrical sensitivity - optimization of thermal experiment - identification.

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On the basis of nonlinear heat condaction equations and UBP some mathematical models of technological processes of thermal processing of materials are developed. In particular, the mathematical model of temperature process of coal coking is offered with taking into account the features of heating of central part of the coke loading with respect to the diffusion in a steam-gas mixture and the vaporization of moisture, simulated by UBP. The mathematical models of thermal processing of materials in fluidized bed at a level of a separate particle are developed. The modeling of limestone dissociation reaction is executed on the basis of a UBP.

The research of parametrical sensitivity of mathematical models has allowed to establish main laws of changes of sensitivity at various parameters of process. By methods of theory of sensitivity the informational nonequivalent of measurements of temperature in various points of the space area is established. In this connection the problem of optimum planning the thermal experiment for getting inptut data about process with the purpose of identification of external heat transfer parameters is considered.

Thermal experiment is characterized by a structure of a vector of measurements (quantity of parameters and coordinates of points of measurement of temperature) as well as some essential factors of technological character. The optimization means the choice of such conditions of experiment for which an errores of measurements of body's temperature influence in the least degree on accuracy of account of heat transfer parameters.

The error of measurements of temperatures of a body has the value $\Delta T = T^*(x,\tau) - T(x,\tau,\sigma_0)$, where $T^*(x,\tau)$ - is measured temperature, and $T(x,\tau,\sigma_0)$- is true temperature. This error of measurements results in an error of heat transfer coefficient determination on the value $\Delta\sigma = \sigma - \sigma_0$. The linear part of decomposition of body's temperature in the Taylor series on heat transfer coefficient in some its vicinity connects these values by means of the following equality :

$$\Delta\sigma = \Delta T / T_\sigma(,\tau).$$

It follows from this equlity that under increasing of derivative $T_\sigma(,\tau)$ the error of measurements of heated body's temperature decreases its influence on the error of account of value of heat transfer coefficient. As criterion of efficiency of thermal experiment we shall consider the value of average integral sensitivity $F(,\tau_k)$ which we shall define as follows:

$$F(,\tau_k) = \frac{1}{\tau_k} \int_0^{\tau_k} T_\sigma(,\tau) d\tau.$$

The experiment on measurement of temperature should be planned at such significances of parameters of process which would provide the maximum sensitivity of body's temperature with respect to changes of required values. The properties of sensitivity functions are investigated in ranges of values according to conditions of particular technological process. The extremum point of integral criterion of efficiency depends on significances of technological parameters and the character of this dependence is that as well as at a point of the maximum of sensitivity functions.

If we differentiate the initial UBP with respect to the heat transfer parameter of radiation in a boundary condition we shall be able to get possibility of qualitative researches of sensitivity functions. The received boundary - value problem is a problem with a mobile boundary which sharing fields of sensitivity. The movement of the boundary is defined by the initial UBP.

Qualitative and numerical analysis problem shows that the heaviest sensitivity takes place on a surface of a body in initial moment of time then maximum of sensitivity function on space coordinate moves to a centre of the body with decreasing of absolute value. After completion of process of movement of a boundary at further heating of the body the appearance of second extremum of sensitivity function takes place.

The best strategy of measurement of temperature is proposed in view of dynamics of movement of a boundary .

The problems of identification of distributed parameters of external heat transfer at linear and nonlinear boundary conditions of mathematical models of thermal processing are considered. These problems are concerned to inverse boundary-value problems of heat conduction and ill-posed in classical sense.

The methods and algorithms of decision of identification problems using the regularization and functional approximation ideas are suggested. The method of regularization is based on obtaining the decision by minimization of a regulating functional. The regulating functional consists of two integral functionals. The first functional evaluates the value of deviation of the estimated values from the measured ones. The second functional is intended for stabilization of the received decision. The degree of stabilization of the decision is determined by the regularization parameter. The significance of the regularization parameter is determined according to the principle of the residual.

The effective method of decision of identification problems of distributed parameters of external heat transfer is the method polinomial spline - approximation of required functions. The required functions are presented as the splines which are

chosen as the polinomials of the third degree. The identification problem consists of the gradient minimization on basic values of quadratic functional of deviations. The stability of the decision is reached at the expense of the natural step-by-step regularization.

The comparative analysis of complexity of algorithms of various methods is executed. The industrial checking of algorithms of identification of parameters conducted on a experimental data has confirmed the high efficiency of decision of the problem of external heat transfer coefficients determination.