# The speed-up of time integration in industrial multibody system packages

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## 1 Introduction

The simulation of mechanical and mechatronic systems in industrial applications is frequently based on *mechanical multibody system* (MBS) models that describe the real physical system by a finite number of (rigid or elastic) bodies being connected by massless junctions (joints, springs, ...). Typical applications include vehicle system dynamics and robotics.

Based on the principles of classical mechanics the MBS model equations

$$\begin{aligned} M(q)\ddot{q}(t) - f(t,q,\dot{q},\lambda) + G^T(t,q)\lambda &= 0\,, \\ g(t,q) &= 0 \end{aligned} \tag{1}$$

are generated by *multibody formalisms* (see e. g. [6]). In (1) the motion of the MBS is described by $n_q$ position coordinates $q(t)$ that have to satisfy $n_\lambda$ holonomic constraints $g(t,q)=0$. These constraints result from kinematically closed loops in the MBS and from the coupling of substructures that have been modelled independently of each other. Constraint forces $-G^T(t,q)\lambda$ with $G(t,q) := g_q(t,q)$ and $n_\lambda$ Lagrangian multipliers $\lambda$ guarantee that the constraints are always satisfied. All remaining force terms in the MBS are collected in $f(t,q,\dot{q},\lambda)$, such that $f$ may depend on $\lambda$ if the model includes friction forces that depend on the constraint forces $-G^T(t,q)\lambda$. The symmetric mass matrix $M(q)$ is positive definite on $\ker G(t,q)$.

Eqs. (1) may be considered as simplified prototype of model equations in practical applications that contain e. g. typically additional differential and algebraic equations defining internal state variables in force elements and coordinates of contact points. Throughout the paper we assume that the matrix

$$\begin{pmatrix} M(q) & , (t,q,\dot{q},\lambda) \\ G(t,q) & 0 \end{pmatrix} \quad \text{with} \quad , (t,q,\dot{q},\lambda) := -f_\lambda(t,q,\dot{q},\lambda) + G^T(t,q) \tag{2}$$

is non-singular such that (1) forms a differential-algebraic equation (DAE) of index 3 (see e. g. [4, pp. 463ff]). Before discretization the index has to be reduced to guarantee a stable numerical integration. Substituting the original constraints $g(t,q)=0$ by their time derivative

$$0 = \frac{d}{dt} g(t,q(t)) = g_t(t,q) + g_q(t,q)\dot{q}(t) = g_t(t,q) + G(t,q)\dot{q}(t) \tag{3}$$

we get an analytically equivalent index-2 system. Standard techniques like projection steps ([5]) or the Gear–Gupta–Leimkuhler formulation ([4, p. 465]) are used to guarantee that the numerical solution does nevertheless satisfy the constraints $g=0$ in (1).

Standard DAE software like the BDF code DASSL (see e. g. [4, Sec. VII.3]) may be applied to the index-2 system to solve initial value problems for (1) numerically. In practice, however, the "'general purpose"' simulation software has to be adapted to the special strucutre of (1) to reduce the large cpu-time in simulations for MBS with many degrees of freedom and in parameter variations that require the solution of hundreds of initial value problems for (1).

Over the last 10 years an extensive part of the literature on numerical solution of DAEs has been devoted to this specific application. In the present paper we focus on the application of these results in the industrial multibody system simulation package SIMPACK and on the combination of multibody formalisms and numerical methods. The benefits of the new and modified numerical algorithms are illustrated with examples from vehicle industry.

Substantial savings of cpu-time may be achieved e. g. using partitioned integrators for non-stiff MBS (see Sec. 2), exploiting the structure of Jacobian matrices in implicit integrators (see Sec. 3), and using integration software for ordinary differential equations (ODEs) for MBS without holonomic constraints (see Sec. 4).

## 2 Partitioned integrators for constrained mechanical systems

Standard DAE software is based on stiff integrators from ODE theory (BDF, implicit Runge–Kutta methods). Alternatively, partitioned integrators for non-stiff MBS have been developed. These integration methods are based on non-stiff ODE methods (explicit Runge–Kutta and extrapolation methods, Adams methods), they make use of the special structure of the model equations (1) (see [4, Sec. VII.6] for a comprehensive overview and [1] for a summary of some actual results).

The prototype of these integration methods is the *half-explicit Euler method* for the index-2 formulation of (1):

$$\begin{aligned} \frac{q_{n+1}-q_n}{h} - v_n &= 0\,, \\ M(q_n)\frac{v_{n+1}-v_n}{h} - f(t_n,q_n,v_n,\lambda_n) + G^T(t_n,q_n)\lambda_n &= 0\,, \\ g_t(t_{n+1},q_{n+1}) + G(t_{n+1},q_{n+1})v_{n+1} &= 0\,. \end{aligned} \tag{4}$$

Here we have rewritten (1) as first order system with velocities $v := \dot{q}$ , the holonomic constraints have been substituted by (3).

For given $q_n$ and $v_n$ the half-explicit Euler method defines $q_{n+1}$ in an explicit way and $v_{n+1}$, $\lambda_n$ as solution of a system of nonlinear equations with a Jacobian of the form (2) where $(t, q, \dot{q}, \lambda)$ in the upper blocks are substituted by $(t_n, q_n, v_n, \lambda)$ while $(t, q)$ in the lower left block are substituted by $(t_{n+1}, q_{n+1})$.

Various ways to evaluate the Jacobian and to solve the systems of nonlinear equations efficiently have been discussed by Lubich et al. ([5]) who implemented most of these algorithms in their successful extrapolation code MEXX. We point out the problems in solving the systems of nonlinear equations for model equations being generated by the multibody formalisms in SIMPACK and compare algorithms to evaluate the matrices $M$, , , and $G$ in the Jacobian (see (2)).

## 3 The evaluation of Jacobian matrices in the dynamical simulation of stiff MBS

As in ODE theory the model equations (1) for *stiff* MBS have to be solved by stiff integrators (BDF, implicit Runge–Kutta methods). The most simple method of this type is the first order backward Euler method that is again applied to the index-2 formulation of the model equations:

$$\begin{aligned} \frac{q_{n+1}-q_n}{h} - v_{n+1} &= 0\,, \\ M(q_{n+1})\frac{v_{n+1}-v_n}{h} - f(t_{n+1},q_{n+1},v_{n+1},\lambda_{n+1}) + G^T(t_{n+1},q_{n+1})\lambda_{n+1} &= 0\,, \\ g_t(t_{n+1},q_{n+1}) + G(t_{n+1},q_{n+1})v_{n+1} &= 0 \end{aligned} \tag{5}$$

that defines $(q_{n+1}, v_{n+1}, \lambda_{n+1})$ as solution of a system of nonlinear equations. The Jacobian $J$ of this system has the typical structure

$$J(q, v, \lambda; h) := \frac{1}{h} \cdot \text{blockdiag}\,(\, I\,,\; M(q)\,,\; 0\,) + \tilde{J}(q, v, \lambda)$$

with a matrix $\tilde{J}$ being independent of $h$. The evaluation of the Jacobian $J$ causes often more than 80% of the overall numerical effort in practical applications.

We discuss the implementation and the benefits of 2 modifications in the evaluation of the Jacobian that have been proposed by Doc. Dr. C. Führer (Lund University, Sweden):

1. All standard implicit integrators from ODE and DAE theory are implemented such that the approximation of $J$ is kept fixed over several time steps. After a change of $h$ cpu-time may be saved if additionally an old approximation of $\tilde{J}$ is used in the re-evaluation of $J$.

2. In larger models the sparsity structure of $J$ may be exploited to compute a finite difference approximation of $J \in \mathbb{R}^{N \times N}$ with substantially less than $N + 1$ function calls ([2]).

## 4 Numerical aspects in the comparison of multibody formalisms

There is a great variety of multibody formalisms to generate the MBS model equations. SIMPACK offers the Residual formalism ([3]) resulting in (1) and an explicit MBS formalism that gives model equations

$$\begin{aligned} \ddot{q}(t) &= M^{-1}(q)(f(t, q, \dot{q}, \lambda) - G^T(t, q)\lambda)\,, \\ 0 &= g(t, q)\,. \end{aligned} \tag{6}$$

Obviously, the explicit formalism has advantages for MBS *without* constraints since any ODE solver may be applied to (6) in this case. On the other hand the residual

$$M(q)\ddot{q} - f(t, q, \dot{q}, \lambda) + G^T(t, q)\lambda$$

in (1) may be evaluated by a factor $2 \ldots 4$ faster than the right hand side in (6).

Surprisingly, the explicit formalism was superior to the Residual formalism in various applications even for MBS *with* constraints (that enforce the use of a DAE solver in both cases). We explain this phenomenon by a convergence analysis of Newton's method applied to the corrector equations in stiff ODE and DAE integrators.

## References


[1] M. Arnold and A. Murua. Non-stiff integrators for differential-algebraic systems of index 2. submitted to: *Annals of Numerical Mathematics*, 1997.

[2] T.F. Coleman, B.S. Garbow, and J.J. Moré. Software for estimating sparse Jacobian matrices. *ACM Transactions on Mathematical Software*, 10:329–345, 1984.

[3] A. Eichberger. *Simulation von Mehrkörpersystemen auf parallelen Rechnerarchitekturen.* Fortschritt-Berichte VDI Reihe 8, Nr. 332. VDI–Verlag, Düsseldorf, 1993.

[4] E. Hairer and G. Wanner. *Solving Ordinary Differential Equations. II. Stiff and Differential-Algebraic Problems.* Springer-Verlag, Berlin Heidelberg New York, 2nd edition, 1996.

[5] Ch. Lubich, Ch. Engstler, U. Nowak, and U. Pöhle. Numerical integration of constrained mechanical systems using MEXX. *Mech. Struct. Mach.*, 23:473–495, 1995.

[6] R.E. Roberson and R. Schwertassek. *Dynamics of Multibody Systems.* Springer-Verlag, Berlin Heidelberg New York, 1988.