Simulation of the Filtering Processes in Layered Strata

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A sufficiently wide class of various filtering processes in reservoirs are described mathematically by partial differential equations containing a term (terms) of diffusion (or heat conduction) type. The following processes can be rated in this class:

-pressure fields - both for the incompressible and for compressible liquids;

-concentration fields - when displacement of mixing liquids occurs (with sorption and without it);

-temperature fields, etc.[1],[2].

The consideration of the mentioned processes in the strata of a layered structure entails the necessity to formulate the so-called conjugation conditions on the contact boundaries of adjacent layeres.

They are identical for all mentioned processes. More exactly, if U is the sought-for function (the pressure, concentration or temperature), G is a boundary of two adjacent layers and k^+ (k^-) denotes the corresponding physical characteristics (coefficient of filtration, diffusion or heat conduction) to the right (left) side from the boundary, then on G the following two continuity conditions have to be fulfilled [3]:

$$U^- = U^+, \qquad k^- \frac{\partial U^+}{\partial n} = k^+ \frac{\partial U^+}{\partial n}. \tag{1}$$

These conditions may lead to some computation problems due to two reasons. First, there is rather a considerably difference between coefficients k^+ and k^-. Second, the characteristical thickness of the layers is small compared with their longitudinal sizes. To avoid the mentioned difficulties, one usually proceeds from the initial two- or three-dimensional model with conditions (1) to a one- or two-dimensional model perfoming, in one way or another, the averaging in the layer thickness [3].

One of the authors has introduced for making the averaging procedure accurate a special integral paraboloc spline. In [4],[5] this spline was proved to be efficient when solving some filtration problems. We will describe the essence of the proposed method by the following model problem.

Suppose that $\overrightarrow{x'} \in R^{n+1}$ and $\overrightarrow{x'} = (\overrightarrow{x}, z)$, where $\overrightarrow{x} \subset D \in R^n$ and z denotes the coordinate directed along the normal to the planes G_i, $i=\overline{1,N}$ that divide the layered stratum into separate layeres. Suppose, next, that the process in the $i-$th separate layer is described by the equation:

$$div(k_i\ \overrightarrow{grad}\ U_i)+\frac{\partial}{\partial z}(k_{i,z}\frac{\partial U_i}{\partial z})+L_i(U_i)=0, \qquad i=\overline{0,N}, \tag{2}$$

Here the operator L_i describes the convection, sorption, the time dependence of the process and so on. To Eq.(2) there should be added boundary conditions for external boundaries G_0, G_{N+1} of the extreme layers, those for boundary D as well as the initial conditions and, if necessary, some complementary differential equations (e.g. for the sorption kinetics).

We will introduce now a function for the $i-$th layer averaged in thickness as

$$u_i(\overrightarrow{x})=H_i^{-1}\int_{z_i}^{z_{i+1}}U_i(\overrightarrow{x},z)dz, \qquad i=\overline{0,N}. \tag{3}$$

Then the spline of the second order can be defined uniequelly from conditions (1) on the boundaries $G_i, i=\overline{1,N}$, from the boundary conditions on boundaries G_0, G_{N+1} and conservation conditions (3) ([4],[5]). Thus from problem (2) in R^{N+1} we proceed to the system of the equations in R^N:

$$div(k_i\ \overrightarrow{grad}\ u_i)+L_i(u_i)+\sum_{j=0}^{N}\beta_{ij}u_j+\phi_i=0, \qquad i=\overline{0,N}, \tag{4}$$

In (4) the coefficients β_i depend both on the physical constant $k_{i,z}$ and on the thickness H_i of the layers; ϕ_i depends on the right sides of the boundary conditions on G_0, G_{N+1}.

The method proved to be efficient for a number of filtration theory problems.

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