Bifurcation theory methods in the problemabout capillary-gravity waves on the interface on two-fluids flow

Loginov B.V., Ulyanovsk State Technical University 432027 Ulyanovsk, Severny Venets str.32,root@ulgpu.simbirsk.su

Trenogin V.A., Moskow Institute of Steel and Alloys 117936, Moskow, Lenin prospect 4,vladilen@trenogin.misa.ac.ru

Bifurcation theory under group symmetry conditions and its interaction with group analysis of differential equations methods [1-4] allow to investigate free boundary value problems of capillary-gravity waves in fluid layers [5,6]. Here we consider the problem about waves on the interface of two nonmixing fluids flow in spatial layer [7] having geophysical and technical applications. Corresponding planar problem was considered in [8].

Potential flows of two nonmixing incompressible fluids with densities ρ_1 and ρ_2,bifurcating from the flows with constant velocities V_1 and V_2 in the Ox-axis direction is describing by the following system in dimensionless variables

$$\Delta\Phi_1 = 0, \qquad -1 < z < f(x,y); \quad \Delta\Phi_2 = 0, \qquad f(x,y) < z < k$$

.

$$\frac{\partial\Phi_1}{\partial z}|_{z=-1} = 0; \frac{\partial\Phi_2}{\partial z}|_{z=k} = 0;$$

$$\frac{\partial\Phi_j}{\partial z} - \frac{\partial f}{\partial x} - \frac{\partial\Phi_j}{\partial x}\frac{\partial f}{\partial x} - \frac{\partial\Phi_j}{\partial y}\frac{\partial f}{\partial y} = 0, \quad z = f(x,y), \quad j = 1,2$$

$$\frac{\partial\Phi_1}{\partial x} - \tilde{k}_0\frac{\partial\Phi_2}{\partial x} + \frac{1}{2}|\nabla\Phi_1|^2 - \frac{\tilde{k}_0}{2}|\nabla\Phi_2|^2 - (1-k_0)F^2 f = \gamma F^2[\frac{\partial}{\partial x}(\frac{f_x}{\sqrt{1+{f_x}^2+{f_y}^2}}) +$$

$$+\frac{\partial}{\partial y}(\frac{f_y}{\sqrt{1+f_x^2+f_y^2}})], \quad z = f(x,y)$$

Here $\Phi_j(x,y,z) = -V_j x + \varphi_j(x,y,z)$ - the potentials of fluids velocities, $z = f(x,y)$ the interface fluids surface close to horizontal plane $z = 0$, $\quad k = \frac{h_2}{h_1}$- the ratio of fluid layers thicknesses,$k_0 = \frac{\rho_2}{\rho_1}$, $\quad \tilde{k}_0 = \frac{V_2^2}{V_1^2}k_0$, $\quad F = \frac{\sqrt{h_1 g}}{V_1}$ is the inverse to Froud number magnitude, $\gamma = \frac{\sigma}{\rho_1 h_1^2 g}$ the Bond number and g the acceleration of gravity. It is posed the problem of periodical solutions construction with the periods $\frac{2\pi}{a} = a_1$ and $\frac{2\pi}{b} = b_1$ along the coordinate axes, and $\prod_0$ is the rectangle of periodicity.

After straightening of the free boundary

$$\zeta = \frac{(k+1)(z - f(x,y))}{(k - 2f(x,y) - 1)z + 2k + (k-1)f(x,y)}$$

$$u_j(x,y,\zeta) = \Phi_j(x,y,-\frac{(2k + kf - f)\zeta + (k+1)f}{(k - 2f - 1)\zeta - k - 1})$$

setting $F^2 = F^2_{m,n} + \varepsilon$ we obtain nonlinear system linear part of which represents Fredholm operator $B : C^{2+\alpha}([-1,0] \times \prod_0) \dot{+} C^{2+\alpha}([0,1] \times \prod_0) \dot{+} C^{2+\alpha}(\prod_0) \to C^{\alpha}([-1,0] \times \prod_0) \dot{+} C^{\alpha}([0,1] \times \prod_0) \dot{+} C^{\alpha}(\prod_0)$. The case $k = \frac{h_2}{h_1} = 1$ is considered separately because of the impossiblity of the direct substitution $k = 1$ in calculated formulae. The bifurcating points F^2_{mn} are determined by the dispersion relation connecting the natural numbers m, n and periods a_1, b_1 with parameters $F^2$and γ. It is shown that 2,4,6,8,10,12- multiple degenerations of the operator B are possible. However the regular hexagonal and octagonal lattices of periodicity are possible when more heavy fluid is setting over (physically impossible case).

By group analysis methods the bifurcation equations for these cases are constructed and asymptotics of bifurcating solutions are calculated.

References

1 Loginov, B.V.: Branching Theory of Solutions of Nonlinear Equations under Group Invariance Conditions.Tashkent,"Fan"-Verlag,1985-184p(Russian)

2 Loginov, B.V.: On the construction of the general form of branching equation by its group summetry EQUADIFF-VII. Enlarged Abstracts. Praha 1989,48-50.

3 Loginov, B.V.: Group analysis methods for construction and investigation of the bifurcation equation. Applications of Mathematics 37(1992),N4,241-248

4 Ovsyannikov, L.V.: Group Analysis of Differential Equations. Moskwa,Nauka 1978-400p;AP,New York 1982.

5 Loginov, B.V., Kuznetsov, A.O.: Capillary-gravity waves over the flat surface. European Journal of Mechanics B/Fluids,15(1996),2,259-280.

6 Loginov, B.V., Karpova, S.A.,Trenogin, V.A.: Bifurcation, symmetry and parameter continuation in some problems about capillary-gravity waves. Progress in Industrial Mathematics at ECMI-96,B.G.Teubner,Stuttgart,1997,432-439.

7 Loginov, B.V.,Trofimov E.V.: The calculation of capillary-gravity waves asymptotics on the interface of two fluids of finite depth. In "Differential Equations of Mathematical Physics and their Applications". Tashkent,"Fan"-Verlag(1989), 57-66.

8 Kochin, N.E.: Determination regoureuse des ondes permanentes d'ampleur finie a la surface de separation de deux liquides de profondeur finie. Math.Annallen, 98(1928),582-615.