Analysis of the Dynamics of Dispersed Phase Systems

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Abstract

Dispersed phase systems are rather common and important in industrial applications. In some cases they still deserve considerable theoretical attention since the practical realization of the relevant technology is only partially fulfilled due to the difficulty to predict the asymptotic behavior of these dispersions. We mention just one industrial application which, being still challenging despite its economical importance, has been focusing the attention of petroleum engineers for a long time. Some off-shore crude oil reservoirs suffer from marine water filtration through the floor of the sea, so that the extracted primary product is a dispersion of oil in water. In some cases gases may also be present as a third component. If the off-shore extraction platform is sufficiently close to the coast, it would be less expensive and more efficient to pipeline the product directly to a treatment plant on the coast rather than to ship it by tankers. In this connection it is well known that a dispersion of oil in water is less viscous, and, therefore, more easily pumpable, than crude oil alone. The main difficulty is to maintain the initial dispersion. The original one is usually "optimized" by a jet-pump which realizes the required size distribution of droplets. However dispersions are not naturally stable. The question is if and how long this dispersion will not start to break up into separated phases. Indeed once separation begins, oil, being lighter,

tends to migrate towards the top of the pipeline leaving the original dispersion below it. It may also happen that this separation process shows up through more than a single free boundary. Depending on the extension of the pipeline internal boundary wet by the oil phase, the viscosity of the bulk fluid may grow significantly up to the impossibility to continue pumping.

As a first step to get some insight within this problem, we propose a model for the dynamics of breakage and coalescence of droplets. This model is not completely new (see [**?**]); however our approach is rather different. Once such a dynamics is fully understood, the subsequent problem will be a free-boundary problem in which the mechanism of phase separation is controlled by the underlying dynamics of droplets collision.

For simplicity we consider only the spatially homogeneous case. The initial value problem for the droplet size distribution function $f(v,t)$ (for unit volume of dispersion) is

$$\begin{cases} \dfrac{\partial f}{\partial t} & = \frac{1}{2}\displaystyle\int_0^v P(v',v-v',t)\ dv' - \int_0^{v_{max}} P(v,v^m e,t)\ dv' \\ f(v,0) & = f_o(v) \end{cases} \tag{1}$$

where v stays for the droplet volume, f_o is given and P is a symmetric function of (v,v') proportional to

$$(v^{1/3}+(v')^{1/3})^2 f(v,t)f(v',t) \tag{2}$$

The proportionality factorλ is a known function of the physical state of the whole system which should measure the "efficency"of coalescence. Indeed not all collisions result in coalescence, and therefore we may expect λ to depend on (v,v') as well on temperature and local value of the shear stress. Therefore the whole problem requires the coupling of (**??**) with the Navier-Stokes and energy equations. Equation (**??**) is quite naturally suggested by the Boltzmann equation for gas dynamics.

The whole problem appears to be rather complex. Here we present some preliminary results for the the simpler case in which collisions are not driven by shearing motion, the temperature is uniform, and λ is a constant. The following consistency results are easily proved:

(i) $\frac{d}{dt}\int_0^{v_{max}} f\ dv < 0$ (the number of droplets in the dispersion decays with time),

(ii) $\frac{d}{dt}\int_0^{v_{max}} vf\ dv = 0$ (conservation of total droplets volume),

(iii) $f(v,t) \geq 0$ for all $v \geq 0$ and $t \geq 0$.

We also can prove that (**??**) has *one and only one solution* which turns out to be global in time. As a consequence we obtain the following results concerning the asymptotic behavior of f:

(iv) If $\gamma(\widetilde{v},\widetilde{t}) = 0$ is the *locus* , of maxima of f, then $(\partial f/\partial t)|_\Gamma < 0$. This means, in particular, that $\sup_{v\in(0,v_{max})} f \leq \sup_{v\in(0,v_{max})} f_o$ for all $t \geq 0$ and that $\lim_{t\to\infty} f(v,t) = 0$

The investigation of droplet dynamics under shearing condition is actually under investigation. A careful analysis is needed as far as the coalescence efficency is concerned. Indeed we assumed, for simplicity, that this parameter is a constant. It is evident that to fully characterize the dynamics one must consider some parameter as a *characteristic collision time* and this in turn has to depend on shearing situation, density and viscosity of the dispersed and continuous phase. Moreover droplets which are in an agglomerated state may coalesce or not, depending on several factors: for example we expect coalescence if the thin film of continuous phase separating two colliding droplets has drained sufficiently to allow collapse of the phase boundary. On the other hand, local eddies may impart sufficient kinetic energy to the droplets to prevent coalescence when this energy is greater of the energy of adhesion of the droplet pair. We also expect that there is some *minimum drop size* above which turbulent velocity fluctuations may prevent coalescence of agglomerated drops. Finally this minimum should actually depend on shear, density, viscosity and, possibly, on geometry.

References

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