A mathematical model of water filtration through a layer of ground coffee

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Abstract

A theoretical and experimental research program aimed at investigating the complex processes taking place during the filtration of water through ground coffee in the expresso-coffee machine was promoted years ago by illycaffè s.p.a. (Trieste, Italy).

A first series of tests and experiments ([1]) exhibited some non-standard aspects of the phenomena occurring in the operation of the espresso-coffee machine as, for instance, the non applicability of Darcy's law. A second relevant feature of such filtration processes is the mechanical and chemical interaction between the flow and the porous matrix. In particular, a migration of particles removed by the medium and an accumulation of them in proximity to the outflow surface have been observed.

Due to the relevant complexity, the mathematical investigation has been carried out by examining separately the various aspects of the problem. In the model presented in [3] a single family of particles is considered. The removed particles accumulate and give rise to the formation of a compact layer with high hydraulic resistance. In [2], [5] several species of particles have been introduced in the model; however, they are allowed to flow out of the system. In [2] the particles are transported convectively by the flux and a combined effect of mass removal and friction of the flow on the solid grains has been considered. On the other hand, in [5] the species are transported either as solid particles or as solutes.

The model presented in [4] is aimed at combining the models mentioned above, removing some simplifying assumptions as, for istance, the choice of a uniform and constant porosity and initial distribution of particles made in [3].

The main features of the mathematical model formulated in [4] are the following:

(i) the removed components are both sufficiently fine solid particles, which are transported convectively by the flow, and substances which may also diffuse in the liquid (typically as solutes),

(ii) the porosity is affected by the removal process, but here we neglect additional effects such as mechanical compression of the porous matrix by the

flow,

(iii) the removal rate depends on the concentration of the particles still bound to the porous matrix and on the liquid flux intensity,

(iv) the hydraulic permeability in the part of the layer where the removal process takes place depends on the concentration of each species and on the porosity, while it is much lower and constant in the compact layer,

(v) the fine particles do not leave the system but they accumulate in the vicinity of the outflow surface, giving rise to a compact layer, whose structure depends on the history of the flow,

(vi) effects of gravity and interdiffusion can be neglected.

In [4] the mathematical investigation of the model $(i) - (vi)$ is performed. Existence and uniqueness for the solution of the system corresponding to the model are proved. At the same time, some relevant qualitative properties of the solution are shown.

References

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