Integrating Ordinary Differential Equations on Manifolds*

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Abstract

During the last four decades, a vast amount of research has been put into time-integration of ordinary differential equations. In the beginning of the period, a large number of methods were constructed, and they were often optimized in a general sense, for large classes of applications. However, for long-time integration, most of the methods gave unsatisfactory results. In the eighties, integration of Hamiltonian problems with symplectic integrators became the focus of attention. In two dimensions, symplecticity is equivalent to area-preservation, and it is easily shown experimentally that failure in preservation of area will lead to solutions that are not only qualitatively but also quantitatively incorrect for long-time integration.

Symplecticity implies a quadratic constraint on the configuration space, and it can be turned into a relation to be satisfied by the coefficients of the numerical scheme. All consistent Runge-Kutta methods preserve linear and some of them also preserve quadratic constraints. However, no Runge-Kutta method satisfies cubic or higher order constraints.

Since the beginning of the nineties, different approaches, often termed "geometric integration", have been pursued by some authors. The idea is to integrate differential equations in such a way that the configuration space of the model problem is respected by the numerical solution. The common underlying theme for all the geometric integrators is the concept of coordinate transformations. Instead of solving the problem directly as posed on the nonlinear configuration space, we apply some transformations that bring the problem into a linear space where we may apply classical methods and still preserve the invariants.

In this talk we explain the concept of geometric integration in terms of coordinate transformations. We review some of the recently developed methods, e.g. [Crouch & Grossman (J. Nonlin. Sci. 3, 1–33), Iserles & Nørsett (Tech. Rep. 1997/NA3, DAMTP, Cambridge), Munthe-Kaas (Tech. Rep. 1997/NA14,

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DAMTP, Cambridge)], and provide numerical simulations that illustrate gains and possible drawbacks connected to these techniques.