Uncontrollable Inaccuracy in Inverse Problems

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Abstract

In this paper the influence of errors of function derivatives which have been measured by experiment (uncontrollable inaccuracy) on the results of inverse problem solution was investigated. It was shown that these errors distort the solution of inverse problem quality near the beginning of interval where the solution to be analyzed . Several methods intended to remove the influence of uncontrollable inaccuracy have been suggested . In particular the method of special filtration of initial data of inverse problem was described .

1 Introduction

The inaccuracy is inevitable in experimental measurings of physical values. It consist of inaccuracy of measuring instruments , noise value and inaccuracy of visual means. The value of this inaccuracy can been evaluated by technical indicators of measuring instruments. They does not exceed 5-10 percents as a rule.

The experimental measurings are chosen as initial data for following calculations with the use of mathematical models in many practical important problems. For example, the inverse problems for evolutionary processes [1,2] , the control problems with the use of experimental data [3,4] belong to this class. The values of function $\tilde{x}(t)$, measured in experiment and the values of their derivatives of various orders in initial moment $(t_0 = 0)$ $\dot{\tilde{x}}(0), \ddot{\tilde{x}}(0), ...$ are present in these problems. The order of derivatives is rising with the rise of order of differential equations system which described the motion of real object. But it is impossible in principle to evaluate the inaccuracy of these derivatives and they are usually chosen to be of infinite value. The indicated inaccuracy was called the uncontrollable inaccuracy [5,6].

2 The statement of a Problem

As an example let us consider the inverse problem of unbalance evaluation of deformable rotor characteristics which has two supports [5]. The physical significance of symbols and parameters in equations will not be interpreted. The main purpose is simply to preserve the structure of expressions.

The rotor motion is described by the system of ordinary differential equations of 18th order [5] . The equation for determination of unknown functions (characteristics of unbalance) $z_1(t), z_2(t), z_3(t)$ has a form

$$\int_0^t (t-\tau) z_i(\tau) d\tau = u_i(t), \qquad (i = 1, 2, 3), \qquad (1)$$

where $u_i(t)$ are the known functions, obtained from experiment. For example, the function $u_2(t)$ has the form:

$$u_2(t) = \int_0^t \{[\sum_{j=1}^{4}(t-\tau)^{j-1}N_j^A]\ddot{\eta}_A(\tau) + [\sum_{j=1}^{4}(t-\tau)^{j-1}N_j^A]\ddot{\eta}_B(\tau)\}d\tau +$$

$$+N_5^A\ddot{\eta}_A(t) + N_5^B\ddot{\eta}_B(t) + N_6 + N_7 t + N_8 t^2 + N_9 t^3;$$

where $N_1^A, N_2^A, N_3^A, N_4^A, N_1^B, N_2^B, N_3^B, N_4^B, N_5^A, N_5^B$ are constants,

$$N_6 = K_1\ddot{\eta}_A(0) + K_2\ddot{\eta}_B(0), N_7 = K_3\eta_A^{(3)}(0) + K_4\ddot{\eta}_A(0) + K_5\eta_B^{(3)}(0) + K_6\ddot{\eta}_B(0),$$

$$N_8 = K_7\dot{\eta}_A(0) + K_8\dot{\eta}_B(0) + K_9\eta_A(0) + K_{10}\eta_B(0) + K_{11}; N_9 = K_{12}\dot{\eta}_A(0) + K_{13}\dot{\eta}_B(0),$$

$K_1 - K_{13}$ are constants; $\ddot{\eta}_A(t), \ddot{\eta}_B(t)$ are the experimental data reflecting the vibration of rotor supports in vertical direction. It is assumed that the errors of values $\Delta\eta_A^3, \Delta\eta_B^3, \Delta\eta_A^2, \Delta\eta_B^2, \Delta\eta_A^1,$ $\Delta\eta_B^1, \Delta\eta_A, \Delta\eta_B$ appeared when values of $\eta_A^{(3)}(0), \eta_B^{(3)}(0), \ddot{\eta}_A(0), \ddot{\eta}_B(0), \dot{\eta}_A(0), \dot{\eta}_B(0), \eta_A(0), \eta_B(0)$ are measured.

We will determine the influence of errors upon the solution of inverse problem (1).The right part of equation (1) represents the outlet of initial system $\ddot{\eta}_A(t), \ddot{\eta}_B(t)$ initiated by unknown impact $z_2(t)$ only. We exclude all the rest actions and initial conditions. The uncontrollable inaccuracy in initial conditions leads to appearance of the supplementary terms $\mu_i y_i(t)$ in the expression for $u_2(t)$, where $y_i(t)$ is the solution of homogeneous initial system $z_1(t) = z_2(t) = z_3(t) \equiv 0$ with nonzero initial conditions, μ_i - const.

Using the equations of motion we obtain the linear expression with regard to $\ddot{\eta}_A(t)$ and $\ddot{\eta}_B(t)$:

$$\gamma_{14}\eta_A^{(4)}(t) + \gamma_{13}\eta_A^{(3)}(t) + \gamma_{12}\ddot{\eta}_A(t) + \gamma_{11}\dot{\eta}_A(t) + \gamma_{10}\eta_A(t) + \gamma_{24}\eta_B^{(4)}(t) + \gamma_{23}\eta_B^{(3)}(t) +$$

$$+\gamma_{22}\ddot{\eta}_B(t) + \gamma_{21}\dot{\eta}_B(t) + \gamma_{20}\eta_B(t) + \gamma_{42} = \tilde{z}_2(t), \tag{2}$$

where γ_{ik} are constant.

Let us consider two functions $\ddot{\eta}_A(t)$ and $\ddot{\eta}_B(t)$ which satisfy (2) identically when $\tilde{z}_2(t) \equiv 0$ for $t > 0$ and which satisfy the zero initial conditions:

$$\eta_A^{(3)}(0) = \eta_B^{(3)}(0) = \ddot{\eta}_A(0) = \ddot{\eta}_B(0) = \dot{\eta}_A(0) = \dot{\eta}_B(0) = \eta_A(0) = \eta_B(0) \equiv 0. \tag{3}$$

Let the functions $\ddot{\tilde{\eta}}_A(t), \ddot{\tilde{\eta}}_B(t)$ coincide with functions $\ddot{\eta}_A(t), \ddot{\eta}_B(t)$ when $t > 0$ and satisfy the initial conditions :

$$\tilde{\eta}_A(0) = \Delta\eta_A, \quad \dot{\tilde{\eta}}_A(0) = \Delta\eta_A^1, \quad \ddot{\tilde{\eta}}_A(0) = \Delta\eta_A^2, \quad \tilde{\eta}_A^{(3)}(0) = \Delta\eta_A^3,$$

$$\tilde{\eta}_B(0) = \Delta\eta_B, \quad \dot{\tilde{\eta}}_B(0) = \Delta\eta_B^1, \quad \ddot{\tilde{\eta}}_B(0) = \Delta\eta_B^2, \quad \tilde{\eta}_B^{(3)}(0) = \Delta\eta_B^3.$$

The function $\tilde{z}_2(t)$ will differ from zero when $\ddot{\tilde{\eta}}_A(t), \ddot{\tilde{\eta}}_B(t)$ are substituted into expression (2). So, it is possible to reduce the investigation of influence of uncontrollable inaccuracy to the analysis of function $\tilde{z}_2(t)$. The coincidence of $\ddot{\eta}_A(t), \ddot{\eta}_B(t)$ and $\ddot{\tilde{\eta}}_A(t), \ddot{\tilde{\eta}}_B(t)$ when $t > 0$ leads to

$$\ddot{\eta}_A(t) = \ddot{\tilde{\eta}}_A(t)\sigma_+(t), \quad \ddot{\eta}_B(t) = \ddot{\tilde{\eta}}_B(t)\sigma_+(t),$$

where $\sigma_+(t)$ is the asymmetric single step-function [7] . By substitution of $\ddot{\tilde{\eta}}_A(t), \ddot{\tilde{\eta}}_B(t)$ into (2), we get

$$\tilde{z}_2(t) = d_1\acute{\delta}_+(t) + d_0\delta_+(t) + c_0 + c_1 t,$$

when $t > 0$, where d_1, d_0, c_0, c_1 are constants, $\delta_+(t)$ is the asymmetric impulse-function [7]. It is easy to show that the function $\tilde{z}_2(t)$ changes essentially the solution of equation (1) .

We obtain the analogous results in the inverse problem of astrodynamics too [2]:

$$\tilde{z}_2(t) = \beta_1\acute{\delta}_+(t) + \beta_0\delta_+(t),$$

β_1, β_2 are constants.

This statement remains true for nonlinear problems but the character of influence can be more complicated.

3 The filtration of initial data

The exclusion of some interval [0,ϵ] (ϵ is the small value) from solution where it do not true is the single way to remove the influence of this inaccuracy .

Moreover , where it is possible , it is necessary to set the initial condition for the solution of inverse problem to correspond the state of rest . Then the all items which determine uncontrollable inaccuracy ought to be set equal to zero according to physical sense .

The following method of influence removal of uncontrollable inaccuracy on result of inverse problem solution is suggested: the items which determine the uncontrollable values of initial conditions are excluded from function $u_2(t)$ in equation (1) by means of the special filtration [6].

The components of $a_1 t$ kind (a_1 is constant) are excluded from function $u_2(t)$ in problem of unbalance identification because the errors of very these terms can not be evaluated . The function $u_2(t)$ is defined on interval [0,T] and $u_2(0) = 0$.This function is continued on interval [-T,0] by odd way . Here we used the properties of Leganders polynomial [7] . Let us define the value of a_1 from expression

$$a_1 = \frac{3}{2T^3} \int_{-T}^{T} t u_2(t) dt.$$

Then the filtered function $u_2^f(t)$ $\quad (u_2^f(t) = u_2(t) - a_1 t)$ is substituted into right-hand side of integral equation of inverse problem instead of $u_2(t)$.

The test of numerical computations demonstrate the ample efficiency of suggested method. The solution of equation of kind (1) by regularization method (suggested by A.N.Tikhonov) with the selection of regularization parameter by help of discrepancy method is understood as numerical computation in this case. It was problematic to analyze the influence of uncontrollable inaccuracy by numerical computation . We may fix only the general tendency: the numerical approximate solution is distorted on the part of interval [0,T] equals 0.2T ; the deviation of this solution from exact solution may be great in uniform metric . The approximate solution distorts smaller in uniform metric if the value of inaccuracy of initial data δ is rising .

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