Investigation of the Faraday resonance in water waves at nearly critical depths with the aid of computer algebra

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We consider the two-dimensional wave motion of an ideal inviscid fluid contained in a rectangular basin with vertical side walls which oscillates in vertical direction according to a harmonic law. The nonlinear Faraday resonance (the subharmonic excitation of gravitational surface waves of finite amplitude in a vertically oscillating basin) is studied in the case that the fluid depth is equal or close to a critical depth (the depth at which the third-order nonlinear correction to the wave frequency is zero).

A similar problem for unforced nonlinear standing water waves was solved in [1–3]. Tadjbakhsh and Keller [1] found that the frequency-amplitude dependences are qualitatively different for fluid depths above and below a certain depth, which is called the critical one by Miles and Henderson [4]. For the three-dimensional problem, the same result follows from the formulas obtained by Ya. I. Sekerj-Zenkovitch [2] and by Verma and Keller [3]. All these results were obtained in the framework of the third-order asymptotic theory with a small parameter (amplitude times wave number).

The influence of the fluid depth on the behavior of the subharmonically excited water waves in a vertically oscillating basin was studied, theoretically and experimentally, in [5–8]. In these papers, the resonance curves for several fluid depths were obtained in the framework of the third-order theory. The curves for the depths above and below the critical depth were found to differ qualitatively. If the fluid depth in the basin is above the critical one, then the resonance curves have the “soft-spring” character. If the fluid depth is below the critical one, the theory predicts the “hard-spring” character of the curves. The experimental results of Virnig et al. [7] and Kalinitchenko et al. [8] confirm these properties of the resonance curves for depths above and below the critical depth but not very close to it.

In the present paper, we study the influence of the fluid depth on the behavior of the subharmonically excited water waves in a plane rectangular basin. The dependence of the wave amplitude on the excitation frequency is derived analytically using Maple, a system of symbolic computations.

The Lagrangian formulation is used to write out the exact nonlinear equations and the dynamic and kinematic boundary conditions. An asymptotic procedure based on the Krylov–Bogolyubov averaging method is developed. Considering the case of depths close to the critical value, one has to calculate fifth-order approximations in a small parameter. We suppose that the ratio of wave height to wavelength and the ratio of the maximum acceleration of the basin to the gravitational acceleration are small positive quantities, and there is a relation between the orders of these quantities.

However, this takes an extent of analytical work by orders of a magnitude greater than that performed in [1–3, 5, 6, 8], which is quite tedious in itself. To overcome these difficulties, an application package based on the Maple computer algebra system was developed and used in this study.

The results obtained allowed us to predict the following properties of the resonance curves: (i) if the fluid depth is equal to or greater than the critical depth, then the resonance curve bears the “soft-spring” character; (ii) otherwise, the resonance curve consists of two separate branches; one branch has the “soft-spring” character and the other branch, the “hard-spring” character. This results in the hysteresis effect, which has an unusual form for the parametric resonance.

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