Erosion using powder bombardment

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1 Problem description

In flat screen televisions glass plates are present in which holes are formed with a diameter of 200 μm. To make these holes, a mask is attached on the glass. This mask contains holes of the desired diameter. Then the glass undergoes a powder bombardment, where particles with a diameter up to 20 μm are used. To answer the question whether disturbances in the jet or jet direction are important for the contour or depth of a resulting hole, a model of this process has to be found, which can predict the shape of these contours.

2 Model

We describe the jet of particles by a continuous mass flux S $[kg/m^2 s]$ (see figure 1). Further, the erosion rate E $[kg\ s^{-1}/kg\ s^{-1}]$ is defined as the mass of eroded material in gram per time unit divided by the particle flow rate $[kg/s]$ from the jet. Now the displacement $\eta(x,t)$ of the contour in the direction of the velocity of the particles is given by

$$\frac{\partial \eta}{\partial t} = -\frac{1}{\rho} S E, \tag{2.1}$$

with ρ the density of the glass. The variable x is defined perpendicular to the direction of the velocity of the particles (see figure 1).

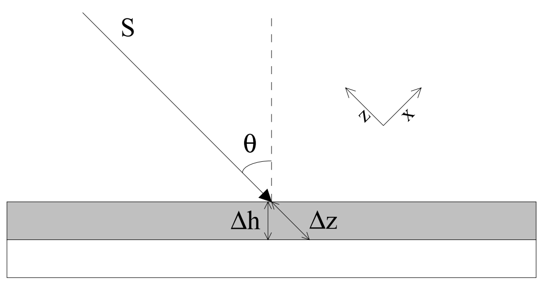


Figure 1: The amount of eroded material is angle dependent.

Experiments were performed to measure the erosion rate E as a function of the initial velocity $\mathbf{v}_i$ of the particles, which resulted in

$$E = c(\mathbf{v}_i, -\mathbf{n})^k = c|\mathbf{v}_i|^k \cos^k \theta, \tag{2.2}$$

where k is an experimental parameter in the range $2 < k < 4$ and θ is the angle between the outward normal $\mathbf{n}$ of the surface and the velocity vector. The constant c $[m/s]^{-k}$ is measured at $1.7 \cdot 10^{-6}$.

However, near the edges of the mask less impacts occur as a result of the size of the particles (see figure 2). To account for this effect, we introduce a position dependent particle mass flux $\phi(x)$ such that

$$\phi(x) = S \text{ for } \delta < x < L - \delta, \tag{2.3}$$

and such that $\phi(0) = \phi(L) = 0$, with L the width of the hole. The size δ of the boundary layer will be of the order of the diameter of the used particles. An advantage of introducing a boundary layer is that by prescribing $\phi(0) = \phi(L) = 0$, the boundary conditions $\eta(x,t) = 0$ at $x = 0$ and $x = L$ are always satisfied. The behavior of $\phi(x)$ in the boundary layer depends on the distribution of the particle size, and is chosen linear.

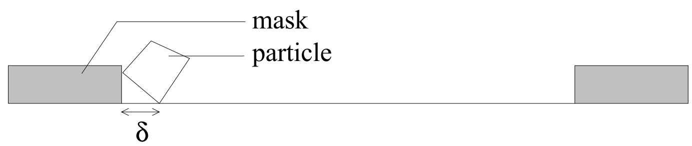


Figure 2: Particles remove less material near the edges as a result of their size.

Equations (2.1), (2.2) and (2.3) can be nondimensionalised using typical values for width (L), flux (S) and time ($\rho L/(c|\mathbf{v}_i|^k S)$), to obtain

$$\frac{\partial \eta(x,t)}{\partial t} = -\phi(x)\cos^k\theta, \tag{2.4}$$

with boundary conditions $\eta(0,t) = \eta(1,t) = 0$.

3 Numerical solution

By introducing $q = \partial\eta/\partial t \equiv \eta_t$ and $p = \partial\eta/\partial x \equiv \eta_x$, we can rewrite (2.4) into

$$q + \phi(x)E(p) = 0, \tag{3.5}$$

with $E(p) = (1+p^2)^{-\frac{k}{2}}$, which can be shown from (2.2) and $p = \tan\theta$. By differentiating both sides of (3.5) with respect to x we find

$$p_t + (\phi(x)E(p))_x = 0, \tag{3.6}$$

which is a conservation law for the slope p. The characteristics of this equation from the left and right boundary layer eventually cross each other. Therefore we will not numerically exploit the characteristic equations.

For $\delta \le x \le 1-\delta$ the problem is of the form

$$\frac{\partial p}{\partial t} + \frac{\partial f}{\partial x} = 0, \tag{3.7}$$

with $f = f(p)$. The boundary conditions must be found from the equations in the boundary layer. Further, the numerical scheme used must remain stable when discontinuities in p occur. We have chosen to solve this problem numerically using the method of Roe. This method takes into account the characteristic wave speed, and can handle discontinuities due to the crossing characteristics.

The derivative p in the boundary layer can be calculated by the method of characteristics. The characteristic equation in the boundary layers is given by

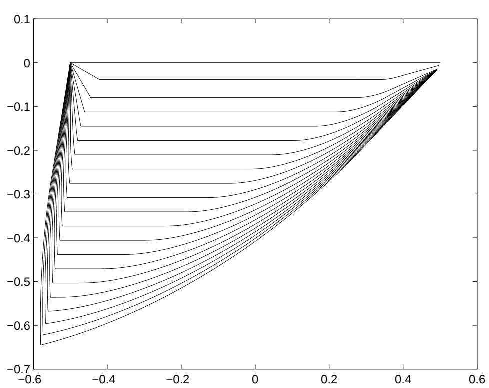
$$\frac{dp}{dt} = \pm\frac{1}{\delta}E(p), \tag{3.8}$$

thus $p = p(t)$. This equation is solved using the trapezoidal rule. Thus combining the method of Roe with the method of characteristics we can compute (3.6) on the whole interval.

4 Results

An algorithm was designed which makes it possible to bombard with an arbitrary angle on an arbitrary contour. It is based on the fact that in the model the gap grows in the direction of the particle velocity. This makes it possible to sequentially bombard the glass with jets having different angles.

Figure 3 shows the result of a bombardment under an angle of 30^o (left). Due to the first order scheme an error occurs in the right-top of the picture. Further it is clear that there is a discontinuity in the derivative of the contour. Right the result of a second bombardment with an angle of 30^o is given, after a vertical bombardment has occurred.



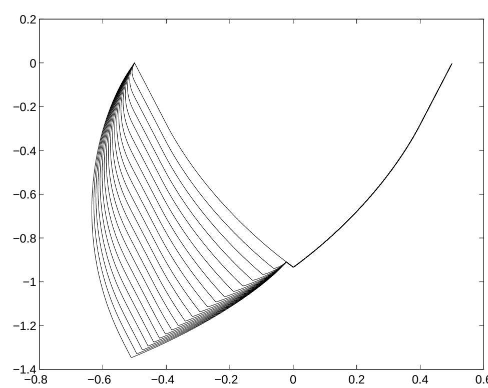


Figure 3: Left the depth as a function of position when shooting on a straight curve under an angle of 30^o. Right the depth as a function of position when shooting first from above on a straight curve for 1 time unit and then shooting on this curve under an angle of 30^o. Used parameters are $\delta = 0.1$, $t_{end} = 1$, number of points in time 2000, in position 1000. Curves are displayed every 0.05 unit of time.

At first sight these results are according to earlier measurements. However, more detailed validation of the model has to take place.