Spectral Utilization in Next Generation Wireless Digital Data Applications
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Summary

Next generation cellular systems will carry digital data traffic just like wireless LAN's: internet access, browsing and speech, MPEG video, file transfers, control data, multimedia traffic, commercial transactions etc. which are primarily characterized by very irregular data flows. The radio resource, whether mobile, wireless LAN, or fixed wireless access has a strictly limited bandwidth which is normally much smaller than coax and optical lines and forms a bottleneck for bursty data. To smooth the data flow we can use leaky bucket buffering to limit peak data flow at the expense of introducing time delay and possible buffer overflow and consequent data loss. This paper discusses statistical techniques adapted from digital network analysis which allows analysis of these problems. Using a multiclass Markov fluid model for various classes of data source we can derive exact probability distributions of buffer occupancy (equivalent to time delay), buffer overflow, and relate these measures to the bandwidth provided in the radio channel. Finally using an equivalent bandwidth concept we can extend the analysis to multiplexed data on a radio channel and also bound the performance.

Data Traffic characteristics and modelling

Digital networks and radio channels carrying digital data are hard to utilize fully because of the highly variable and sporadic nature of much digital traffic as noted above. The cellular radio resource used for communication with mobile users, and wireless LANs in offices has especially restricted capacity and the problem presents itself in the most extreme form here.
Data flow can be characterized by a number of parameters such as peak to mean bit rate ratio, probability distributions of bit rates but these fail to capture the dynamics of the burstyness. A stochastic model, which can be configured to resemble a number of non-periodic sources, is the Markov fluid. Here the data source is an n-state discrete Markov process which, in state i, emits data at a constant bit rate h_i and the state transition probabilities per unit time from state i to j are defined in the matrix Q_{ij}.
For such a source, fed to a leaky bucket buffer with output rate c, let $P_i(x)$ be the joint probability that the source is in state i *and* that the buffer content is less than x. The Markov fluid evolution is described by the following differential

equation [1], [2]

$$\frac{\partial P_i(t,x)}{\partial t} + (h_i - c)\frac{\partial P_i(t,x)}{\partial x} = \sum_j Q_{ji} P_j(t,x) \tag{1}$$

If $\lambda_{mean} < c$ then the system is ergodic as $t \to \infty$ and defining $\mathbf{F}(x) = P(\infty, x)$, eqn (1) reduces to

$$(\mathbf{h}_i^{-1} - c\mathbf{I})\frac{\partial \mathbf{F}(\mathbf{x})}{\partial x} = \mathbf{Q}^T \mathbf{F}(x) \tag{2}$$

where $\mathbf{h} = \mathrm{diag}(h_0, h_1, ...)$. To solve this linear pde we find the RHS eigenvectors ϕ_i and eigenvalues λ_i of the matrix $\mathbf{h}^{-1}\mathbf{Q}^T$. The eigenvalues λ_i are sorted into three sets, those which are negative, $i \in i-$; those which are positive, $i \in i+$; and the single zero one, $i \in i0$. We know that $\mathbf{F}(\infty)$ is the vector of equilibrium state probabilities and the general solution to (2) is

$$\mathbf{F}(x) = \mathbf{F}(\infty) + \sum_{i \in i-} a_i \phi_i \exp(\lambda_i x) \tag{3}$$

There is actually sufficient information to solve for the coefficients a_i. Given ergodic behaviour then m, the number of negative eigenvalues of $\mathbf{h}^{-1}\mathbf{Q}^T$ and hence the number of non-zero coefficients a_{i-} is equal to the number of states with flows $h_i > c$ and for each of the m such flows there is a corresponding boundary condition for the vector $\mathbf{F}$:

$$h_i > c \Rightarrow \mathbf{F}_i(0) = 0 \tag{4}$$

Finally there is the single linear constraint $\mathbf{F}(\infty)^T \mathbf{1} = 1$ where $\mathbf{1}$ is the vector containing all ones. The probability function of buffer contents $g(x)$ is given by

$$\begin{aligned} g(x) &= 1 - \mathbf{1}^T \mathbf{F}(x) \\ &= \mathbf{1}^T \sum_{i \in i-} a_i \phi_i \exp(\lambda_i x) \end{aligned}$$

Anick [1] approximates $g(x)$ using the most-positive negative eigenvalue and this leads to a simple exponential distribution, good for large x:

$$g(x) \approx a_{max} \exp(\lambda_{max} x) \mathbf{1}^T \phi_{max} \tag{5}$$

Using these formulas we can get exact expressions or approximations for the distribution of data in the buffer and the probability of overflow if the buffer is finite. At a given radio bandwidth, this allows us to compute the distribution function of data lag in the buffer and whether the digital quality of service (QOS) is adequate.

Effective Bandwidths

The solution above is exact but it only applies to a single source. When several Markov fluid sources are multiplexed there are no exact solutions to the buffer overflow probability, however bounds can be found for the negative eigenvalue with smallest absolute value of the equivalent transition matrix and effective bandwidth methods can be developed. The effective bandwidth of a total data load $W[0,t]$ on the interval $[0,t]$ is defined as

$$\alpha(s,t) = \frac{1}{st}\log(\mathbf{E}e^{W[0,t]}) \tag{6}$$

and can be computed for a number of data source models. For example the effective bandwidth for the Markov fluid source is

$$\alpha(s,t) = \frac{1}{st}\log(\mathbf{1}^T\exp[(\mathbf{Q}^T+\mathbf{h}s)t]\pi) \tag{7}$$

where π is the vector of equilibrium state probabilities If N data flows with effective bandwidths $B_1,..,B_N$ are multiplexed, the resulting effective bandwidth is the sum of the individual EBW's.

$$B_{eff} = B_1 + B_2 + ... + B_N \tag{8}$$

Effective bandwidths, with some difficulty, give lower bounds on the largest negative eigenvalue in the overall transition matrix for multiplexed sources in eqn (5) [2]. Thus we can use effective bandwidths to lower-bound the quality of service for a wireless resource carrying multiplexed digital traffic.

Quality of service and Spectrum Efficiency

A number of typical data source are analyzed and the effect of buffering on wireless performance is examined, both for single and multiplexed sources, in terms of the trade-off between buffer behaviour and radio bandwidth. The spectral efficiency of the radio link: peak/average bit rate is also examined. If the peak flow rate is made small the data flow is highly smoothed and radio spectrum is conserved but buffer time delays sometimes become quite long and this may be unacceptible for some classes of data.

[1] Anick, Mitra and Sondhi: "Stochastic theory of Data-handling system with Multiple Sources", BSTJ 61(8), pp.1871-1894
[2] Kesidis et.al.: "Effective Bandwidths for Multiclass Markov Fluids", IEEE/ACM Trans. Networking, 1(4), Aug. 1994, pp.424-428