Circuit simulation in consideration of electronic noise

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Progress in today's high-technology industries is strongly associated with the development of new mathematical tools. A typical illustration of this partnership is the mathematical modeling and numerical simulation of electronic circuits and semiconductor devices. Circuit simulation is a standard task for the computer-aided design of electronic circuits, especially in the field of integrated circuits (e. g., memory chips). It saves time in the development and money with respect to the construction of prototypes.

Up to now stochastic disturbances have not been considered in the nonlinear transient model and simulation of electronic circuits. Because of the reduction resp. the integration of electronic circuits profitting signals get close to disturbing signals such as electronic noise. We can distinguish between thermal noise, shot noise and flicker noise which have all their reasons in the semiconductor material [1].

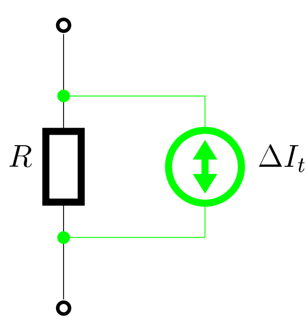

Figure 1: Resistor model under consideration of thermal noise.

These noise sources can be modeled as current sources with randomly distributed current supply. This current sources are shunted to the devices in which the noise causes occure.

The stochastic behavior of the noise sources are modeled by stochastic processes. Most of the processes can be thought as white noise W_t, with statistics depending on the regarding noise source. E. g., the noise current ΔI_t of a thermal noise source in a semiconductor device can be modeled as $\Delta I_t = \sqrt{\frac{4kT}{R}\Delta f} \cdot W_t$, where k is the Boltzmann constant, T the temperature, R the resistance of the device, and Δf the frequency bandwith of the device noise.

Fig. 1 shows the circuit diagramm of a resistor including thermal noise. The noise source with current supply ΔI_t is shunted to the resistor device R.

The modeling of an integrated circuit including noise sources results in a system of differential-algebraic equations, where a stochastic addend is attached, i. e.

$$C(X_t) \cdot \dot{X}_t + f(X_t) - g(t) + G(X_t) \cdot W_t = 0,$$

with the stochastic process X_t containing the searched voltages and currents of the circuit. The matrix function C collects the dynamic devices, the vector function f the static devices and the vector function g the independent voltage sources [2]. The matrix function G does not explicitly depend on the simulation time t. It is a sparse matrix of dimension $n \times m$, where n is the dimension of the solution vector function X_t and m the number of noise sources in the circuit (i. g. $m \gg n$). The random attains in this model via the vector function W_t, a multidimensional Gaussian white noise process. In this paper Itô's interpretation [5] of the circuit equations will be used.

In the area of stochastic differential equations we have to determine in which criteria for the solution of a SDE we are interested [4]. In this paper we will examine the pathwise solution of a SDE. That means we fix the random part in the stochastic process white noise. We get an implementation of the white noise and solve the SDE for this special implementation.

Numerical pathwise approximation schemes for SDEs $X_t = X_0 + \int_{t_0}^t a(s, X_s)\, ds + \int_{t_0}^t b(s, X_s)\, dB_s$, often need partial derivatives $\frac{\partial a}{\partial x}$, $\frac{\partial b}{\partial x}$, and more than one function evaluation of a and b in each integration step. This should be avoided in circuit simulation because of the extensive drift a and diffusion b.

A new multistep method of strong convergence order 1.0 will be presented in this paper. It uses only one function evaluation of a and b in each integration step and it avoids the analytical calculation of partial derivatives. The scheme, here called PEN22, can in the 1-dimensional case be written as

$$X_{n+1} = X_n + a_n h + (a_n - a_{n-1})\frac{1}{2}h + b_n \frac{a_n - a_{n-1}}{X_n - X_{n-1}}\Big(I_{(1,0)} - \Delta B_n \frac{1}{2}h\Big) + \\ + b_n \Delta B_{n+1} + (b_n - b_{n-1})\frac{I_{(0,1)}}{h} + b_n \frac{b_n - b_{n-1}}{X_n - X_{n-1}}\Big(I_{(1,1)} - \Delta B_n \frac{I_{(0,1)}}{h}\Big)$$

where $t_0, \ldots, t_n, \ldots, t_{\text{end}}$ is the discretization of the simulation interval, $h := t_{n+1} - t_n$ the (constant) step size, $\Delta B_{n+1} := B_{t_{n+1}} - B_{t_n}$ the Brownian increment, X_n the numerical approximation of X_{t_n}, $a_n := a(t_n, X_n)$ resp. $b_n := b(t_n, X_n)$ the function evaluations of a resp. b, $I_{(j,k)} := I_{(j,k),t_n,t_{n+1}}$ the multiple Itô-integral with $j, k = 0, 1$. Itô-Taylor-serieses for a and b have been the ansatz functions to develop this scheme.

The new scheme was compared to the explicit order 1.0 strong scheme [4], called EO1SS in this paper, $X_{n+1} = X_n + a_n h + b_n \Delta B_{n+1} + \frac{1}{\sqrt{h}}(b(t_n, \chi) - b_n) I_{(1,1)}$, where $\chi = X_n + a_n h + b_n \sqrt{h}$ (1-dimensional case).

To show that the new scheme has strong order 1.0 the geometrical Brownian motion $\dot{X}_t = \alpha X_t + \beta X_t \cdot W_t$, $X_{t_0} = 1$, $\alpha, \beta \in \mathbb{R}$, has been simulated in the interval $[0, 1]$, cf. tab. 1. The given results for EO1SS and PEN22 show the same order of strong convergence for both methods.

Stepsize h	2^{-6}	2^{-7}	2^{-8}	2^{-9}	2^{-10}	2^{-11}
1 Path, $\alpha = 1$, $\beta = 1$						
Absolute error EO1SS	3.79	1.19	2.12	0.66	0.51	0.69
Absolute error PEN22	3.77	1.20	2.11	0.66	0.51	0.69
100 Pathes, $\alpha = 1$, $\beta = 1$						
Mean absolute error EO1SS	2.10	2.14	2.36	2.06	1.96	2.01
Mean absolute error PEN22	2.11	2.13	2.35	2.05	1.96	2.01
1 Path, $\alpha = 1.5$, $\beta = 1$						
Absolute error EO1SS	0.1792	0.0214	0.0374	0.0053	0.0024	0.0010
Absolute error PEN22	0.1551	0.0383	0.0151	0.0008	0.0006	0.0002

Table 1: Computational results with strong order schemes EO1SS and PEN22 for the test example geometrical Brownian motion.

To show that the new scheme PEN22 is more efficient than standard schemes a ringoscillator was simulated. This benchmark in circuit simulation [3] is built of n inverters and each inverter contains 3 noise sources. The mathematical modeling yields a n-dimensional system of stochastic differential equations. Tab. 2 shows that PEN22 is about 4 times faster than EO1SS for a ringoscillator with $n = 5$ inverters. Increasing the number n of inverters enlarges further the difference in computation time of both methods.

Stepsize h	10^{-10}	10^{-11}	10^{-12}
CPU-time PEN22	0.44 s	0.87 s	8.66 s
CPU-time EO1SS	0.56 s	3.25 s	32.53 s

Table 2: Computational time for a ringoscillator with $n = 5$ single inverters. The simulation interval was $[0, 2.5 \cdot 10^{-8}]$.

References

[1] M. Cooke: *Halbleiterbauelemente*, Prentice-Hall, Inc., 1990.

[2] U. Feldmann, U. A. Wever, Q. Zheng, R. Schultz and H. Wriedt: *Algorithms for modern circuit simulation*, AEÜ, 46 (1992), p. 274-285.

[3] W. Kampowsky, P. Rentrop and W. Schmidt: *Classification and numerical simulation of electric circuits*, Surv. Math. Ind., 2 (1992), p. 23-65.

[4] P. E. Kloeden and E. Platen: *Numerical Solution of Stochastic Differential Equations*, Applications of Mathematics, Vol. 23, Springer Verlag, 1992.

[5] B. Øksendal: *Stochastic Differential Equations*, 4. Edition, Springer-Verlag, 1995.