THE MOVEMENT OF THE STICKY INCOMPRESSIBLE LIQUID BETWEEN TWO MOVING PARALLEL DISKS

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The movement of the sticky incompressible liquid between two parallel disks, moving towards each other or in the opposite direction, is considered. The below description of possible movement conditions is based on the exact solution of Navier-Stokes equations. The movement stability is analyzed with different initial perturbations given.

There is a large class of processes which can be considered from the mathematical point of view as the movement of liquid between two parallel disks, moving towards each other or in the opposite directions. Here are included such processes as the movement of underground waters, the movement of liquid through the hydraulic pump. These problems are interesting, because some of their solutions, though analytically obtained, can be proved by experiment. For example, let us put two parallel disks in the water and start moving them towards each other and then in the opposite directions. Even with a qualitative assessment we will see that in the first situation (when the disks are approaching each other) the approach force is smaller than the separation force in the second situation (when the disks are moving apart). This can be explained by the different character of the liquid movement: when the disks are approaching the movement is potential, when the disks are moving apart the movement is rotational.

This work deal with a description of the types of possible unstability of the above movements.

To carry out a accurate analysis let us consider the movement of sticky incompressible liquid induced by two parallel disks moving towards each other. Let us assume that the horizontal velocity does not depend on the vertical coordinate while the vertical velocity depends linearly on the distance between the disks. In this case Navier-Stokes equation has the following form [1-3]:

$$\begin{aligned}
&\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=2q\\
&\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial P}{\partial x}+\eta\Delta u \qquad (1)\\
&\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial P}{\partial y}+\eta\Delta v
\end{aligned}$$

where the velocity components are represented as:

$v_x = u(x, y, t),\ v_y = v(x, y, t),\ v_z = -2qz,\ p = p(x, y, t)$

(p - pressure divided by liquid density, q - the constant which defines the disks approach velocity).

For convenience of analysis let us select the potential component from the velocity horizontal components and introduce the flow function:

$$u = qx + \frac{\partial \psi}{\partial y}, \quad v = qy - \frac{\partial \psi}{\partial x}, \tag{2}$$

where ψ - the flow function [1] .

After exclusion of the pressure and introduction of the vorticity ω the movement equations have the following form:

$$\frac{\partial \omega}{\partial t} + \{\psi, \omega\} = -q\left(\frac{\partial}{\partial y}(y \cdot \omega) + \frac{\partial}{\partial x}(x \cdot \omega)\right) + \nu\Delta\omega \tag{3}$$

$$\omega = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = \Delta \psi \tag{4}$$

One of the solutions for the equation (3) is $\psi = 0$, which corresponds to the liquid potential movement, which is known as the movement near the stagnant place. To investigate the stability of this solution let us consider the periodical one-dimensional perturbation. Expressed by the following equation:

$$\psi = k^{-2} \cdot A(t)\cos(k(t) \cdot x) \tag{5}$$

and analyze the change of the vorticity ω in the course of time.

Substituting ψ in the equation (3) with the equation (5) and equating groups of items with the same x powers we will get the following system of nonlinear equations

$$\frac{\partial A}{\partial t} = -qA - \nu A k^2 \qquad \frac{\partial k}{\partial t} = -qk \tag{6}$$

These equations disintegrate into successively solvable linear equations, which allows us to find the general solution:

$$A = \beta \exp\left(-qt + \frac{\nu\alpha^2}{2q} e^{-2qt}\right)$$

$$k = \alpha \exp(-qt), \tag{7}$$

where α, β are free constants, determining the amplitude and the wavelength of the initial perturbation.

The sign q in the equation (7) determines the stability of the solution $\psi = 0$. In this case if q >0, the solution is stable, the amplitude is decreasing; otherwise, the solution is unstable, the amplitude is increasing. However, the solution is unstable only until $t = \frac{1}{2q} \ln \frac{q}{\nu \beta^2}$,after which the amplitude decreases rapidly, owing to dissipation.

The case when $\psi = \sum_{i=1}^{N} A_i(t) \cos(k_i(t) x_i)$, our view, is a matter of particular interest. This perturbation of flow function corresponds to the change of the vorticity ω which has the following form:

$$\omega = -\sum_{i=1}^{N} B_i^2 \exp(-\frac{\eta B_i^2}{2q} e^{2qt}) \cos(k_i(t) x_i) \quad (8)$$

If $q > 0$, the solution is stable, with both the amplitude and the wave number k decreasing in the course of time. Otherwise if $q < 0$, the solution is unstable. However, the increase of the amplitude takes place until $t = \sum_{i=1}^{N} \frac{1}{2q} \ln \frac{q}{\nu B_i^2}$, after which owing to dissipation the amplitude decreases rapidly. As for the wave number k, it is increasing in the course of time. The new and interesting fact which has been discovered in the course of research is that the wave number k, corresponding to the time $t = \sum_{i=1}^{N} \frac{1}{2q} \ln \frac{q}{\nu B_i^2}$, is not dependent on the initial conditions and is equal to $k = \sqrt{\frac{-q}{\nu}}$.

It should be noted that in each of the investigated cases $q > 0$ corresponds to the situation when disks are moving towards each other and $q < 0$ - to the situation when the disks are moving apart.

Conclusions:

1. The analytical solution of Navier-Stokes equation for the movement of incompressible liquid between two disks moving towards each other or in the opposite directions has been obtained.
2. The investigation of the potential movement stability for different initial perturbation of flow function ψ has been carried out. It has been proved that with the evolution of perturbation of a certain type, wave structures with different configurations has be formed.

Bibliography:

1. Fletcher C.A.J. Computational Techniques for Fluid Dynamics. Fundamental and General Techniques. Springer-Verlag Berlin Heidelberg, 1988. 504p.
2. Chandrasekhar S. Hydrodynamic and Stability. Dober publications, Inc.. New York, 1997. 654 p.
3. Craik A.D.D. Wave interaction and fluid flows// J.F.M.,1989.V.198 p. 275-293.

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