Analytical and Numerical Solutions for the Heat Transfer in Periodical Systems with Extended Surfaces

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Periodical systems with extended surfaces (fins) are found in various fields of technics. External surfaces of internal combustion engines, radiators, nodes of relay systems, cosmic apparatus and so on, are examples of these systems [1],[2]. The purpose of simulating such a system is optimization of some its geometric and physical parameters. In doing so, it is important to reveal functional dependence of some integral characteristics (such as fin effectivness, fin efficiency, augmentation factor, etc.) on the mentioned geometric and physical characteristics of the wall and the fin [3]. To clear up these functional characteristics it is of value to obtain analytical solutions for the corresponding mathematical models. Such solutions are usually obtainable when considering the one-dimensional approximation: along the $x-$coordinate directed perpendicularly to the wall thickness or, more precise, along the symmetry axis of the fin [3].

Here we consider an analytical solution in the two-dimensional approximation (the second $y-$axis is directed along the external wall surface). That is, the following problem is examined in the two-dimensional area $G = D \cup D_0$, where the wall $D_0 = \{(x, y) : x \in [-\delta, 0],\ y \in [0, 1]\}$, the fin $D = \{(x, y) :\ x \in [0, l], y \in [0, b(x)]\}$. The temperature in an element G of the periodical system must satisfy the basic differential equation

$$div(k\ \overrightarrow{grad}\,U) = L_t[U], \tag{1}$$

the boundary conditions

$$\frac{\partial U}{\partial n} + \beta U = \beta T \tag{2}$$

and the initial conditions in case the right-side operator is nontrivial. For example, if L_t is a differential operator of the second order in time t (i.e. the hyperbolic heat transfer equation is considered), then two initial conditions should be added:

$$U|_{t=0} = U^0(x, y), \qquad \frac{\partial U}{\partial t}|_{t=0} = Q^0(x, y).$$

In boundary conditions β (the *Biot* criterion) and T (the temperature of surroundings) are the function of boundary (x, y). For example,on the symmetry lines, conditions (2) transform into those of symmetry ($\beta = 0$):

$$\frac{\partial U}{\partial y}|_{y=0} = \frac{\partial U}{\partial y}|_{y=1} = 0.$$

The heat conduction coefficients in direction to the wall and the fin may be different. That is why on the contact line between the wall and the fin $x = 0$, the transition conditions should be formulated. For example, in case of the ideal heat contact we have the conjugation conditions:

$$U|_{-0} = U|_{+0}, \qquad k_0 \frac{\partial U}{\partial x}|_{-0} = k \frac{\partial U}{\partial x}|_{+0}. \tag{3}$$

The idea of an approximate analytical method for solving the problem (1)-(2) in the stationary case ($L_t = 0$) is as follows [4]. First, there are introduced integral values averaged in the wall thickness

$$u_0(y) = \delta^{-1} \int_{-\delta}^{0} U(x,y)dx, \tag{4}$$

and in the fin thickness

$$u(x) = b^{-1} \int_{0}^{b} U(x,y)dy. \tag{5}$$

This done, the temperature inside the wall is considered in the following exponential approximation in the $x-$direction:
$U(x,y) = g_0(y) + (e^{-dx} - 1)g_1(y) + (1 - e^{dx})g_2(y)$,
where $g_i(y),\ i = 0,1,2$ are yet unknown functions, while d being an unknown parameter. A similar decomposition but in the $y-$direction is considered for the fin:

$$U(x,y) = f_0(x) + (e^{\rho y} - 1)f_1(x) + (1 - e^{-\rho y})f_2(x).$$

The conditions (2)-(5) allow us to eliminate functions $f_i(x), g_i(y),\ i = 0,1,2,$ by expressing them through average functions $u(x), u_0(x)$. For these functions, using Eq.(1), we can obtain the boundary problems for ordinary differential equations in the stationary case (one-dimensional heat transfer equation in the nonstationary case). Their general solutions contain some free constants that afterwards are determinable from the conjugation conditions for particular solutions.

Thus obtained approximate solutions are compared with the numerical ones for the initial problem (1)-(3).

References

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