CALCULATING CONTACT ZONE PRESSURES FROM STRAIN AND DEFLECTION DATA

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The determination of interface pressures in contact zones between hard and soft bodies is an important engineering problem, examples of such contacts being those due to the human foot, automobile tyres and objects on conveyor belts in robotic handling. The authors are developing a new method of measuring such pressures, which avoids insertion of the measuring device in the contact zone. The measuring device proposed is an instumental "force plate", a nest of closely-spaced parallel aluminium beams of rectangular cross-section, the hard body, on which is placed the soft body. Two approaches to determining the pressure exerted on a single beam of the nest are being investigated. In the first, data from twenty electric resistance strain gauges spaced out on the underside of the beam are used to reconstruct the load, whilst in the second 190 measurements of the deflection of the beam along its length are made by a traversing gap-measuring electronic probe and this data used to reconstruct the load.

The beams of the nest are such that simple beam theory can be applied, so that for a beam of length L and half-depth c the bending moment M is related to the outer surface longitudinal strain ϵ by

$$M = -(EI\epsilon/c),$$

where E is Young's modulus and I is the second moment of area of the beam's cross-section, whilst M is related to the load W per unit length by the equation

$$d^2M/dy^2 = W, \quad 0 \le y \le L, \tag{1}$$

where y is distance along the beam. Further, the deflection V is related to M by the equation

$$M = EId^2V/dy^2, \quad 0 \le y \le L,$$

so that V satisfies the fourth-order equation

$$EId^4V/dy^4 = W, \quad 0 \le y \le L. \tag{2}$$

The beam is simply-supported, so that

$$M = 0,\ V = 0 \quad at\ y = 0,\ L.$$

Scaling by setting $x = L^{-1}y$, $w = L^2W$, $v = EIL^{-2}V$, the beam's scaled length then being unity, and integrating (1) gives

$$\int_0^1 g_s(x,s)\ w(s)\ ds = M(x), \quad 0 \le x \le 1, \tag{3}$$

where the Green's function $g_s(x,s)$ is given by

$$g_s(x,s) = s(x-1), \quad s < x,$$
$$= x(s-1), \quad x < s,$$

whilst integrating (2) gives

$$\int_0^1 g_d(x,s)\, w(s)\, ds = v(x), \quad 0 \le x \le 1, \tag{4}$$

where the Green's function $g_d(x,s)$ is given by

$$g_d(x,s) = 1/6s(1-x)(2x - s^2 - x^2), \quad s < x,$$
$$= 1/6x(1-s)(2s - s^2 - x^2), \quad x < s.$$

The determination of w from either of these Fredholm integral equations of the first kind is an improperly-posed problem, the one for equation (3) of order 2 and for equation (4) of order 4 [1]. A regularization scheme of projection type based on the non-negativeness of the pressure has been developed and is described in a paper to an earlier ECMI conference [2]. In this the pressure w is approximated by a sum of non-negative basis functions, $b_i^n(x)$, $i = 1,\dots,n$, here either Bernstein polynomials or quadratic B-splines, so that

$$w(x) = \sum_{i=1}^{n} a_i\, b_i^n(x), \quad 0 \le x \le 1,$$

the a_i being determined by minimizing

$$\sum_{j=1}^{m} [Z(x_j) - \sum_{i=1}^{n} a_i f_i^n(x_j)]^2,$$

subject to $a_i \ge 0$, $i = 1,\dots,n$, the points x_j, $j = 1,\dots,m$, being those at which measurements are taken, and where for strain data known

$$Z(x_j) = M(x_j), \quad f_i^n(x_j) = \int_0^1 g_s(x,s)\, b_i^n(s)\, ds, \quad j = 1,\dots,m,$$

and, where for deflection data known,

$$Z(x_j) = v(x_j), \quad f_i^n(x_j) = \int_0^1 g_d(x,s)\, b_i^n(s)\, ds, \quad j = 1,\dots,m.$$

The authors first reconstructed known theoretical and experimental loadings on a single beam from the nest using strain data only, the loadings chosen comprising a sinc function superposed on a uniformly distributed load. As is to be expected with improperly-posed problems the decision as to the number of basis functions to use, that is, the choice of the regularization parameter n, is not straightforward. However, the authors have found that by using two distinct sets of basis functions and a variety of tests, described in a recent paper [3], it is possible to pinpoint satisfactory reconstructions of the loading.

They next investigated an unknown loading provided by a rubber-faced rigid steel indentor as typical of the objects that might be placed on the proposed nest. Removing the rubber facing gives a load with only two points of contact on a typical beam of the nest, whilst for a thick soft facing the two contact points can be expected to spread out, the extent depending on the hardness of the rubber. Even in this latter case however the indentor is in contact over only part of the beam.

In the absence of the rubber the reconstructions for both sets of basis functions give strong indications of point loads, though the sharpness of these loads is not as pronounced as might be expected. For the rubber facing present it is found that, as the number of basis functions for either set is increased, the reconstructions exhibit first one hump but then a gradual shift to two. The tests for a good value of n suggest that the double-humped distribution is closest to the original loading.

A finite element analysis of the problem using the package ANSYS has been carried out by one of the authors [4], in which the beam and indentor are treated as linear isotropic elastic materials and the rubber as a Mooney-Rivlin one. This analysis shows the loading to be clearly double-humped but with a much sharper transition to the peaks than is implied by the reconstructions. Fourier analysis of the reconstructions and the finite element loading shows the latter to have much more significant higher components of spatial frequency than the former. This can be accounted for by the band-width limitations of the reconstructions due to the limited number of strain measurements obtained and consequent spacing between them. Since it is not sensible to increase the number of gauge stations and an effective method of measuring deflections at a large number of points is now available, the authors are investigating the reconstruction of loads from deflection measurements on the basis of equation (4). Whilst this represents a more severely ill-posed problem than that for equation (3), much more data is available and the effects of band-width limitation are thus likely to be reduced. Further, shear effects can be introduced using a modification of equation (2) due to Timoshenko. Initial results are encouraging and will be reported on.

References

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