Vortex breakdown in swirling annular jets

Carl Fredrik Stein
Department of Mathematics, Chalmers University of Technology
S-412 96 Göteborg, Sweden
e-mail: stein@math.chalmers.se
tel. +46 31 772 53 44, fax. +46 31 16 19 73

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1 Introduction

To start with let us consider a non-swirling jet emerging from an annular orifice. At some downstream position the velocity profile of the jet will be well approximated by the profiles of Schlichting and Landau, which are treated in any introductory book on fluid mechanics.

Let us now suppose that we give our jet a gradually increasing swirl velocity. Initially, the flow pattern, shown in figure 1(a) will be almost unchanged compared to the non-swirling one. This remains true, until we have reached a certain degree of swirl, but then the entire flow pattern suddenly changes. The resulting flow pattern, which is illustrated in figure 1(b), is characterised by a large and very stable recirculation zone along the symmetry axis. This sudden change of the flow pattern is called vortex breakdown.

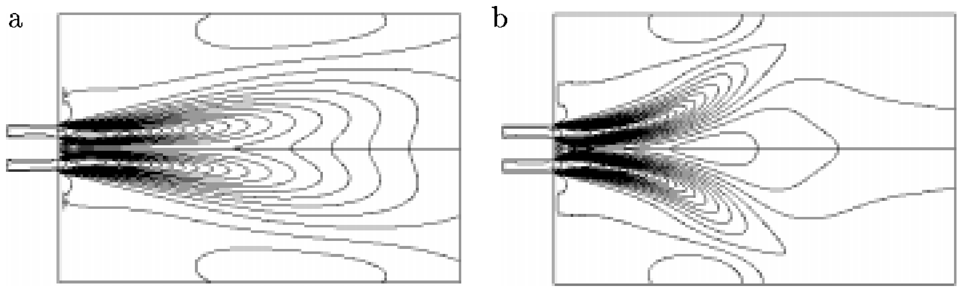

Figure 1: A swirling annular jet (a) before vortex breakdown and (b) after vortex breakdown. The figure shows the level curves for the downstream velocity component of a solution obtained by CFD.

2 The application of swirling jets in combustion chambers

The flow pattern, which is obtained after vortex breakdown is very well suited for flow in for example a combustion chamber, primarily for two reasons: firstly, the large and stable central recirculation zone gives a stable flame and secondly, swirling flow has high turbulence intensity, which accelerates the vapourisation as well as the mixing. These qualities account for the frequent use of swirling jets in the design of low-emission combustion chambers, which is currently undertaken at Volvo Aero Corporation. These small gas turbines, designed for stationary, automotive and marine use are based on LPP (Lean, Pre-mixed, Pre-vapourised) - technology, i.e. air and fuel are mixed and vapourised completely prior to combustion, which yields a very clean flame with little NO_x formation.

For the application of swirling jets in combustion chambers it is an absolute requirement that the flow has undergone vortex breakdown. There is, however, no simple criterion which determines the degree of swirl needed to achieve this. This is elucidated by the fact that quoted critical values in the literature deviate with more than a factor two. To make matters worse, as many as three different dimensionless quantities are commonly used to quantify the degree of swirl. As part of the talk a brief discussion about the appropriateness of the various criteria will be given.

3 Vortex breakdown theory

The most commonly accepted explanation of vortex breakdown is Benjamin's conjugate state theory [1], which proposes that vortex breakdown is a transition between two different swirling flow states. Benjamin [1] also derived his Critical Equation which determines whether vortex breakdown has been undergone or not, by considering whether or not it is possible for extremely long travelling waves to travel upstream. Unfortunately, Benjamin's critical equation is very difficult to extract useful information from unless the velocity profile is uniform. This is a reasonable approximation for channel flow, but hardly for a jet. For more detailed information about Benjamin's and other vortex breakdown theories we refer to [2].

4 Conically self-similar solutions to the Navier-Stokes equations

An alternative way of elucidating vortex breakdown is offered my the conically self-similar solutions to the Navier-Stokes equations. This approach originates from Long [3], and the idea is to seek solutions to the Navier-Stokes equations of the form:

$$\begin{aligned} u_R &= -\frac{\nu\psi'(x)}{R}, \quad u_\theta = -\frac{\nu\psi(x)}{R\sin\theta}, \quad u_\phi = \frac{\nu\Gamma(x)}{R\sin\theta} \\ p - p_\infty &= \frac{\rho\nu^2 q(x)}{R^2}, \quad \Psi = \nu R\psi(x), \quad x = \cos\theta, \end{aligned} \tag{1}$$

where (R,θ,ϕ) are spherical co-ordinates, (u_R,u_θ,u_ϕ) the corresponding velocity components, p the pressure, p_∞ the atmospheric pressure and Ψ a streamfunction. We have also let a prime denote differentiation with respect to x. For (u_R,u_θ,u_ϕ,p) to be a solution to the stationary Navier-Stokes equations ψ and Γ must solve the following system of ODEs:

$$\begin{aligned}
\left(1-x^2\right)\psi' + 2x\psi - \frac{1}{2}\psi^2 &= F\ , && (2)\\
\left(1-x^2\right)F'' + 2xF' - 2F &= \Gamma^2\ , && (3)\\
\left(1-x^2\right)\Gamma'' &= \psi\Gamma'\ , && (4)
\end{aligned}$$

with suitable boundary conditions. Three different kinds of conically self-similar solutions have been identified:

- A near-axis jet. The fluid flows downstream in the vicinity of the symmetry axis, $x = 1$, and upstream close to the cone $x = x_c$.

- A surface jet. The fluid flows downstream along the cone $x = x_c$ and upstream along the symmetry axis.

- A two-cell flow. The fluid flows downstream along some cone $x = x_s$ and upstream both along the cone $x = x_c$ and along the symmetry axis.

Furthermore, asymptotic and numerical studies in [4] indicate that the two last solutions can exist with more rotation than the first one. Therefore, it seems that as the circulation increases over a certain value the near-axis jet breaks up, but this is precisely what happens when vortex breakdown occurs. In the talk we will discuss the relevance of conically self-similar solutions when studying swirling jets.

We will also state an existence, a regularity and a uniqueness result for the singular system of ODE:s (2)-(4), and give an indication of the proofs. Finally we will indicate how these results imply that the existence of a conically self-similar solution to the stationary Navier-Stokes equations implies the existence of a second order swirl-free correction term.

References

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