Random road surfaces and vehicle vibration

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Key words : *random vibrations, weakly correlated random functions, random fields, ordinary differential equations with randomness, Monte-Carlo simulation*

We consider random vibrations of road vehicles excited by random road surfaces. The vehicle is modeled by a multibody system consisting of rigid bodies coupled by springs and dampers. Nonlinear characteristics of springs and dampers are approximated by polynomials. The resulting mathematical model is a system of nonlinear ordinary differential equations of second order

$$\mathbf{A}\mathbf{p}'' + \mathbf{B}\mathbf{p}' + \mathbf{C}\mathbf{p} + \eta\mathbf{D}(\mathbf{p}, \mathbf{p}') = \mathbf{f} \tag{1}$$

with some initial conditions $\mathbf{p}(0) = \mathbf{p}_0$ and $\mathbf{p}'(0) = \mathbf{p}_1$ and a random input term $\mathbf{f} = \mathbf{f}(t, \omega)$ containing the excitation functions. For a n degrees of freedom system the response vector $\mathbf{p} = (p_1, \ldots, p_n)^\tau$ describes the positions of the rigid bodies, $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are real $n \times n$ matrices, $\mathbf{D}(\mathbf{p}, \mathbf{p}')$ is a vector including the nonlinear terms of spring and damper models and η is a perturbation parameter needed in the solution procedure.

The aim of our investigation is the computation of statistical characteristics (as mean, variance, correlation function, spectral density function) of the response variables $p_1, \ldots, p_n$ as well as of their first and second derivatives (velocities, accelerations) for given characteristics of the random input resulting from the random road surface. One method to solve this problem is based on the stochastic analysis of the corresponding representations of solutions of (1). Another method is Monte-Carlo simulation, i.e. solving the system (1) (e.g. numerically) for an ensemble of realizations of the random road and performing statistical estimation procedures for the realizations of $p_i(t, \omega)$. In both cases we need an appropriate mathematical model of the random road surface. It should be simple enough to handle it analytically but not so simple that we can not believe in it as a model of the real world. Further, it should allow an effective generation of samples of road surfaces for Monte-Carlo simulations and for controlling of the input of mechanical experiments.

Our model of the random road surface and subsequently the determination of the desired statistical response characteristics is essentially based on the theory of weakly correlated random functions (see [3]). These functions are characterized by the property that the influence of the random function does not reach far. That means the values of the random function at two points are independent (do not correlate) when the distance of these points exceeds a certain quantity ε. The number ε is called correlation length of the random function. To apply limit theorems ε is assumed to be sufficiently small.

In [1] and [4] one-track excitation of a planar model o a vehicle moving with a constant speed along a straight line has been studied. Thereby the irregularities of the one-dimensional road profile act time-shifted at the several axles of the vehicle. Denoting the random height of the road profile at position x by $g(x,\omega)$ the profile is modeled by

$$g(x,\omega) = \int_{-\infty}^{x} Q(x-u)h_\varepsilon(u,\omega)du,$$

where $h_\varepsilon(.,\omega)$ denotes a weakly stationary and weakly correlated random function of one variable with correlation length ε and $Q(.)$ is some non-random function. The function $Q(.)$ as well as the distribution parameters of h_ε can be determined by comparison with results from measurements. Representations of the response variables of (1) are derived by applying the perturbation method. Approximations of the statistical response characteristics are given in terms of expansions as to the correlation length ε.

This paper presents several extensions of the above results. First, we use 3D-vehicle models which are excited by two parallel tracks of the random road surface (see Fig. 1). The modelling has to take into account that the two tracks are stochastic dependent (correlated). Further, the random road surface is assumed to have a direction-dependent behaviour. We meet these requirements by expressing the two tracks $g_1(x,\omega)$ and $g_2(x,\omega)$ by

$$g_1(x,\omega) = m(x,\omega) + d(x,\omega) \quad \text{and} \quad g_2(x,\omega) = m(x,\omega) - d(x,\omega)$$

using mean and difference profiles m and d where

$$m(x,\omega) = \int_{-\infty}^{x} Q(x-u)h_{1\varepsilon}(u,\omega)du \quad \text{and} \quad d(x,\omega) = \int_{-\infty}^{x} Q(x-u)h_{2\varepsilon}(u,\omega)du$$

with independent and weakly correlated random functions $h_{1\varepsilon}$ and $h_{2\varepsilon}$.

Second, we investigate a model for a complete road surface denoted by $g(x,y,\omega)$ with the transverse position y. It can be used to generate more than two parallel tracks $g_i(x,\omega),\ i = 1,2,\ldots,$ by cross sections of the surface along the lines $y = y_i$ for fixed values y_i, i.e. $g_i(x,\omega) = g(x,y_i,\omega)$. To this end we present a model which is based on the representation

$$g(x,y,\omega) = \int_{-\infty}^{x}\int_{-\infty}^{y} Q(x-u,y-v)h_\varepsilon(u,v,\omega)dudv$$

with a homogeneous and weakly correlated function $h_\varepsilon(.,.,\omega)$ of two variables and some non-random function $Q(.,.)$.

Finally, we present some numerical results concerning the stochastic analysis of response characteristics as well as the generation of samples for Monte-Carlo simulation.

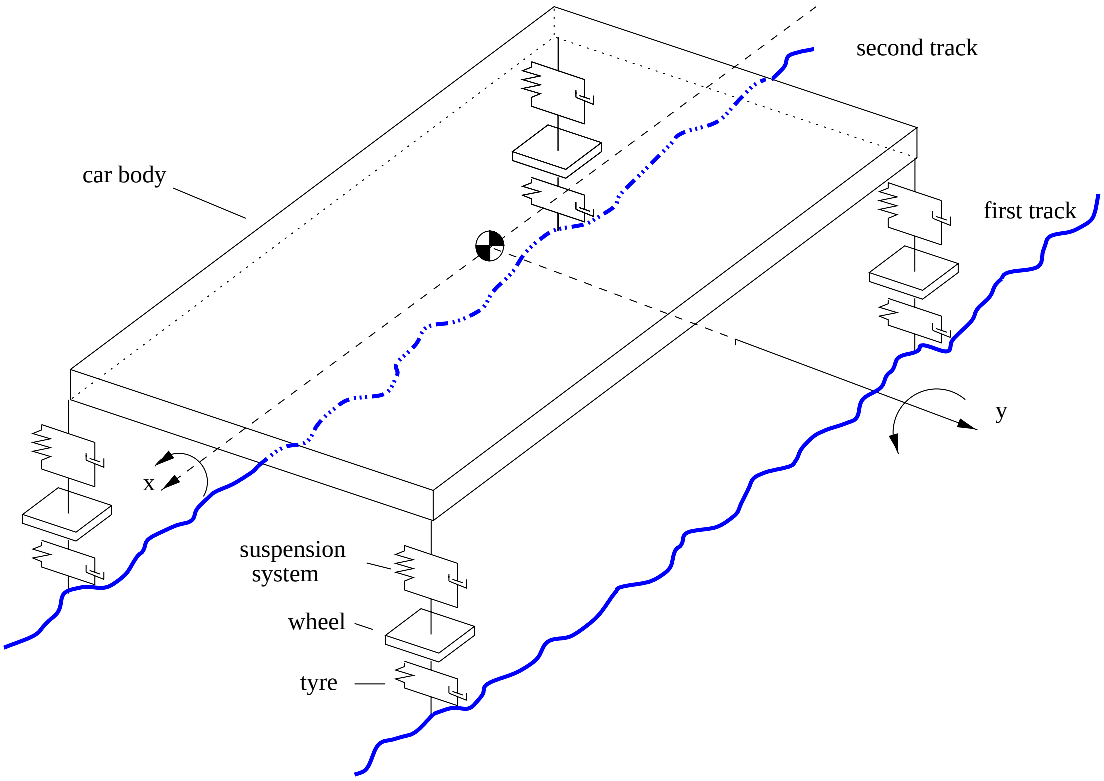

Figure 1: Vehicle model with an excitation by two parallel random tracks

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