Homogeneous growth and population size dependence

Peter Jagers

Dept of Mathematical Statistics, University of Gothenburg, Sweden

Let $m(n)$ be the expected number of children of a mother giving birth when the population size is n — for simplicity think only of splitting populations, so that mothers can give birth only once. Assume that $m(n)$ decreases towards some $m > 1$, as the population grows.
In two quite different setups (the simple Galton-Watson process and a Bell-Anderson cell growth model) with population size dependence, Klebaner and Gyllenberg-Webb, respectively, have found that

$$\sum\{m(n) - m\}/n < \infty$$

guarantees exponential growth.
By a straightforward coupling argument, I (Ann. Appl. Prob. 7, 28–298, 1997) I proved that in general processes with population size dependence

$$\sum\{m(n) - m\} < \infty$$

achieves the same goal. The problem arises, what is general population property securing that the Klebaner-Gyllenberg-Webb condition suffice?
The answer I suggest is *homogeneous growth* in the following sence:
Enter a population at the birth time τ of an individual and consider the expected ratio at some later time t between that individual's progeny size X_t and the total populatioh then, Z_t, , i. e. X_t/Z_t, given the past up to τ. If this does not exceed some constant/ Z_τ, then the population grows homogeneously.