THE MARKOV BRANCHING PROCESS WITH HARSH CATASTROPHES

SUMMARY: It's widely accepted in the ecological literature that the risk of population extinction diminishes as initial numbers increase. As M.L. Rosenzweig puts it, "Rarity begets ultimate rarity". It appears that this is accepted as a verity for models of population growth. Specifically, if X_t is the size at time t of a discrete state stochastic model of population growth, if $T = \inf\{t : X_t = 0\}$ is the extinction time and $P_i(T < \infty) = 1$ ($X_0 = i \geq 1$), then three properties can be identified:

(P1) For any $t > 0$, $P_i(T > t)$ increases with i;

(P2) $P_i(T > t) \uparrow 1$ as $i \uparrow \infty$; and

(P3) P1 and P2 persist after the model has been modified to allow for environmental uncertainty (temporal and catastrophic), if not already included (M.L. Shaffer).

All (subcritical) branching and birth-death processes possess P1, and P2 except in "unlikely" modelling scenarios. I will use the Markov branching process to illustrate ways in which P3 is lost by including environmental effects. This does not happen for temporal uncertainty, but may occur for some model-catastrophes.