Calculus of Variations

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This course will serve as an introduction to the classical theory of the calculus of variations. This theory concerns the extremal problem for functionals of the form

$$E(\gamma) = \int_{t_0}^{t_1} L(t, \gamma(t), \dot{\gamma}(t))dt.$$

Here $\gamma : [t_0, t_1] \to \mathbb{R}^n$ is a curve in $\mathbb{R}^n$ and the Lagrangian L is defined on $\mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n$. The topics to be covered will include the Euler-Lagrange equations and the associated Hamiltonian formalism, convexity theory including the Legendre transform and gauge functions and the Lavrentiev Phenomen. When describing the so-called direct method of the calculus of variations, we will include the Tonelli type of existence results and his theory of the partial regularity of solutions of variational problems. The classical Jacobi theory together with the Weierstrass theory of fields will also be included. Noether's theorem on Lagrangians that are invariant under transformation groups will be discussed together with the inverse problem for the Euler-Lagrange equations. We will cover variational problems with constraints, especially the isoperimetric problems and the holonomic constraints. As an application we will establish the logarithmic Sobolev inequality by Gross. Finally we will give a brief introduction to the theory of non-convex Lagrangians and relaxation.

The lectures will be based on a set of lecture notes that will be available at our website at

http://math.chalmers.se/Research/Geometry Analysis

As a background for the course the standard parts of functional analysis, measure theory and ordinary differential equations will suffice.

The course will consist of 12 lectures and 6 lectures held by Peter Kumlin. We will begin with a short organizational meeting Friday, January 31, at 13.15 in Sal S1.