Svar Matte intro/A BI/KI 16/1 2009

1a) $x^{2-\frac{1}{2}-(3-\frac{2}{3})} = x^{\frac{3}{2}-\frac{7}{3}} = x^{-\frac{5}{6}}$ (1)

1b) $\frac{(x+\sqrt{y})(x-\sqrt{y})}{y(x^2-y)(x-\sqrt{y})} = \frac{1}{y(x-\sqrt{y})}$

1c) $\frac{x(x+1)-2(x-1)}{x(x-1)^2(x+1)} = \frac{x^2-x+2}{x(x-1)^2(x+1)}$

2a) $1-6x+9x^2 = 4x \Leftrightarrow 9x^2-10x+1=0$

$\Leftrightarrow x = \frac{5}{9} \pm \sqrt{\frac{25}{9^2}-\frac{1}{9}} \Leftrightarrow x = \frac{5\pm 4}{9}$ $x=1$ el $x=\frac{1}{9}$

$x=1 \Rightarrow VL<0$

2b) $t \le 1 \Rightarrow 1-t^2 = 1+2t \Leftrightarrow t=0$ el $t=-2$

$t>1 \Rightarrow t^2-1 = 1+2t \Leftrightarrow t = 1\pm\sqrt{3}$

men $1-\sqrt{3}<1$

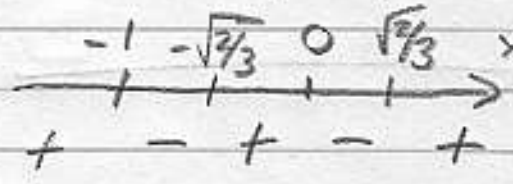
3a) $3(x-\frac{2}{3})(x-1-\sqrt{2})(x-1+\sqrt{2})$

$= 3(x-\frac{2}{3})((x-1)^2-2) = (3x-2)(x^2-2x-1)$

$= 3x^3-8x^2+x+2$

3b) $\frac{x+2(x+1)-3x^2-3x}{x(x+1)} > 0 \Leftrightarrow \frac{-3x^2+2}{x(x+1)} > 0$

$\Leftrightarrow \frac{(x-\sqrt{\frac{2}{3}})(x+\sqrt{\frac{2}{3}})}{x(x+1)} < 0$



4a) $\ln \frac{x^3}{x^{3/2}} = 2 \iff x^{3/2} = e^2$

$\iff x = e^{4/3}$

4b) $e^x - e^{x/2} = 3 - e^x \iff 2e^x - e^{x/2} - 3 = 0$

$e^{x/2} = t \Rightarrow 2t^2 - t - 3 = 0 \iff t = \frac{1}{4} \pm \sqrt{\frac{1}{16} + \frac{3}{2}}$

$e^{x/2} = \frac{1}{4} \pm \frac{5}{4} \iff e^{x/2} = \underbrace{-1}_{<0}$ el. $e^{x/2} = \frac{3}{2}$

$\iff x = 2\ln\frac{3}{2}$

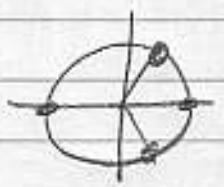
4c) $\ln y = a\ln x + bx$

$\left.\begin{aligned} \ln 1 &= a\ln 2 + 2b \\ \ln e &= a\ln 3 + 3b \end{aligned}\right\} \iff b = -\frac{a\ln 2}{2}$

$1 = a\ln(3/2\sqrt{2})$

$a = \frac{1}{\ln\frac{3}{2\sqrt{2}}} \qquad b = \frac{\ln\frac{1}{\sqrt{2}}}{\ln\frac{3}{2\sqrt{2}}}$

5a) $\sin v = 0$ el. $\cos v = \frac{\sqrt{3}}{2}$



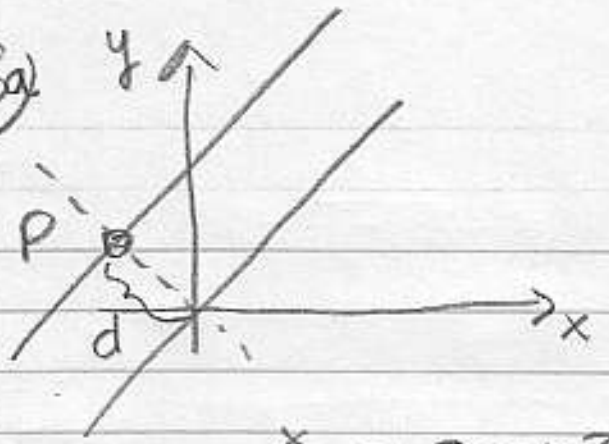
$\iff v = n\pi$ el. $v = \pm\frac{\pi}{3} + n2\pi$

5b) $\cos 5v = -\sin 3v = \sin(-3v) = \cos\left(\frac{\pi}{2} - (-3v)\right)$

$\cos v = \cos\left(\frac{\pi}{2} + 3v\right)$

$\iff \frac{\pi}{2} + 3v = \pm v + n2\pi \iff \begin{matrix} v = -\frac{\pi}{4} + n\pi \\ \text{el. } v = -\frac{\pi}{8} + n\frac{\pi}{2} \end{matrix}$

6a)

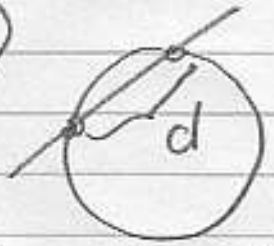


$$P: \begin{cases} y = -\frac{x}{2} \\ y = 2x+3 \end{cases}$$

$$-\frac{x}{2} = 2x+3 \iff x = -\frac{6}{5},\ y = \frac{3}{5}$$

$$d = \frac{3}{5}\sqrt{2^2+1^2} = \frac{3}{5}\sqrt{5}$$

6b)



$$\begin{cases} y = x+1 \\ (x+2)^2+(y-3)^2 = 9 \end{cases}$$

$$(x+2)^2+(x-2)^2 = 9 \iff 2x^2 = 1$$

$$\iff x = \pm\frac{1}{\sqrt{2}},\ y = 1 \pm \frac{1}{\sqrt{2}}$$

$$d = \sqrt{\left(\frac{2}{\sqrt{2}}\right)^2+\left(\frac{2}{\sqrt{2}}\right)^2} = 2$$

7a) tgt $y - y_0 = f'(x_0)(x - x_0)$

$$f'(x) = \frac{\frac{1}{2\sqrt{x+3}}\,x - \sqrt{x+3}}{x^2} = \frac{-x-6}{2x^2\sqrt{x+3}}$$

$$y - 2 = -\frac{7}{4}(x-1) \qquad y = 0 \Rightarrow x = \frac{15}{7}$$

7b) $\sqrt{x+3} = x\sqrt{2} \Rightarrow x+3 = 2x^2$

$$\iff x = \frac{1}{4} \pm \sqrt{\frac{1}{16}+\frac{3}{2}} = \frac{1\pm 5}{4} = \frac{3}{2} \text{ sd. } -1$$

men $x = -1 \Rightarrow HL < 0$