Svar Matte intro/A 24/10-08

(OBS! Ej fullständiga lösn)

1a) $\dfrac{a-2b}{a+2b}$ 1b) $\dfrac{16-2x(x+2)+3(x^2-4)}{x^2-4}$

$$= \frac{x^2-4x+4}{x^2-4} = \frac{x-2}{x+2}$$

2a) $3x^4-8x^2-3=0 \iff x^2 = \frac{4}{3} \pm \sqrt{\frac{16}{9}+1}$

$x^2 = 3$ eller $\underbrace{x^2 = -\frac{1}{3} < 0}_{\text{saknar lösn}}$ $x = \pm\sqrt{3}$

2b) $1-2t+t^2 = 4t^2-12t+9 \iff 3t^2-10t+8=0$

$\iff t = \frac{5}{3} \pm \sqrt{\frac{25}{9}-\frac{8}{3}} = \frac{4}{3}$ eller $\underline{\underline{2}}$

men $2\cdot\frac{4}{3}-3<0$, $2\cdot 2-3>0$

3a) $\dfrac{2x^2-5x+2}{x} \geq 0$ $x = \frac{5}{4} \pm \sqrt{\frac{25}{16}-1}$

$\dfrac{2(x-2)(x-\frac{1}{2})}{x} \geq 0$

$0 \quad 1/2 \quad 2 \quad x$

$- \quad + \quad - \quad +$

$0 < x \leq 1/2$ eller $2 \leq x$

3b) $\dfrac{(x-4)(3x+1)-(x-1)(x-8)}{(x-1)(3x+1)} \geq 0 \iff \dfrac{2x^2-2x-12}{(x-1)(3x+1)} \geq 0$

$x = \frac{1}{2} \pm \sqrt{\frac{1}{4}+6}$ $\dfrac{2(x-3)(x+2)}{(x-1)(3x+1)} \geq 0$

$-2 \quad -1/3 \quad 1 \quad 3 \quad x$

$+ \quad - \quad + \quad - \quad +$

$x \leq -2$ el $-\frac{1}{3} < x < 1$ el. $x \geq 3$

4) Möjliga $\pm(1,2,3,6)$ pröva $\Rightarrow x=-3$

$$
\begin{array}{r l}
 & x^2-4x+2 \\
x+3 \,\big) & x^3-x^2-10x+6 \\
 & -(x^3+3x^2) \\
\hline
 & \quad -4x^2-10x+6 \\
 & -(-4x^2-12x) \\
\hline
 & \qquad 2x+6 \\
 & \quad -(2x+6) \\
\hline
 & \qquad 0
\end{array}
$$

$x^2-4x+2=0$

$x=2\pm\sqrt{4-2}$

faktorisering

$(x+3)(x-2-\sqrt{2})(x-2+\sqrt{2})$

5a) $\ln\left(\dfrac{5\cdot(3x)^3\cdot(3x)^4}{(3x)^6}\right)=2 \iff \ln 15x=2 \iff$

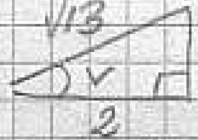
$x=e^2/15$

5b) $\begin{cases} 2=a\ln b \\ 3=a\ln(2b) \end{cases}$ $\quad 3=a(\ln 2+\ln b)=a\ln 2+a\ln b$

$=a\ln 2+2$

$a=\dfrac{1}{\ln 2} \quad \Rightarrow \quad 2\ln 2=\ln b \iff \ln 4=\ln b$

$\iff b=4$

6a) 

$\cos v=2/\sqrt{13}$

6b) $\cos\left(\dfrac{\pi}{2}-\dfrac{v}{2}\right)=\cos v \iff v=\pm\left(\dfrac{\pi}{2}-\dfrac{v}{2}\right)+n\,2\pi$

$\iff \quad v=\dfrac{\pi}{3}+n\dfrac{2\pi}{3}$ el. $v=-\pi+n4\pi$ n heltal

eller dubbelvinkeln $\quad \sin\frac{v}{2}=1-2\sin^2\frac{v}{2}$

7a) $y=-\frac{x}{2}-\frac{1}{4} \Rightarrow k=2 \Rightarrow y-2=2(x+1) \iff y=2x+4$

7b) $2\left((x+1)^2-1\right)+6\left(\left(y-\frac{1}{4}\right)^2-\frac{1}{16}\right)=\frac{5}{8} \iff$

$\iff 2(x+1)^2+6\left(y-\frac{1}{4}\right)^2=3 \iff \dfrac{(x+1)^2}{\left(\sqrt{\frac{3}{2}}\right)^2}+\dfrac{\left(y-\frac{1}{4}\right)^2}{\left(\frac{1}{\sqrt{2}}\right)^2}=1$

7c) $(x-x_0)^2+(y-y_0)^2=r^2$

$(1-x_0)^2+(1-2x_0-3)^2=5$

$\Longleftrightarrow$

$x_0^2-2x_0+1+4x_0^2+8x_0+4=5$

$\Longleftrightarrow$

$5x_0^2+6x_0=0 \Leftrightarrow x_0=0$ el. $x_0=-\frac{6}{5}$

$x^2+(y-3)^2=5$ el. $\left(x+\frac{6}{5}\right)^2+\left(y-\frac{3}{5}\right)^2=5$

8a) $f'(x)=\left(\frac{x^2-1}{3}\right)^3+x\cdot 3\left(\frac{x^2-1}{3}\right)^2\cdot\frac{2x}{3}$

$=\left(\frac{x^2-1}{3}\right)^2\cdot\left(\frac{x^2-1}{3}+2x^2\right)=\left(\frac{x^2-1}{3}\right)^2\cdot\frac{7x^2-1}{3}$

$f'(2)=9 \quad f(2)=2$

tgt $y-2=9(x-2)$ norm $y-2=-\frac{1}{9}(x-2)$

$y=9x-16$ $y=-\frac{x}{9}+\frac{20}{9}$

8b) $f'(x)=0 \Leftrightarrow x=\pm 1,\ x=\pm\frac{1}{\sqrt{7}}$