Svar Matematik del B 1/9-2006

3 KI) $\left(\frac{\sqrt{3}+j}{j-1}\right)^6 = \left(\frac{2e^{j\pi/6}}{\sqrt{2}e^{j\frac{3\pi}{4}}}\right)^6 = \left(2^{1/2}e^{j\left(\frac{\pi}{6}-\frac{3\pi}{4}\right)}\right)^6$

$= \left(2^{1/2}e^{j\left(-\frac{\pi}{4}\right)}\right)^6 = 2^3 e^{j\left(-\frac{3\pi}{2}\right)}$

$= 8j$

3Ö)

$$R = \begin{bmatrix} 3 & 3a+2 & 3b+1 \\ 2 & 2a+2 & 2b-1 \\ 6 & 6a & 6b+2 \end{bmatrix}$$

$\quad\quad\quad$ (1) $\quad$ (2) $\quad$ (3)

(1) $\perp$ (2) $\Leftrightarrow$ $3(3a+2)+2(2a+2)+6\cdot 6a = 0 \Leftrightarrow a = \frac{-10}{49}$

(1) $\perp$ (3) $\Leftrightarrow$ $3(3b+1)+2(2b-1)+6(6b+2) = 0 \Leftrightarrow b = \frac{-1}{49}$

4)

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 2 & 1 & 0 & 1 \\ p & 2 & -1 & 7 \end{array}\right] \begin{array}{c} (-2)\ (-p) \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & -1 & -6 & -3 \\ 0 & 2-p & -1-3p & 7-2p \end{array}\right] (2-p)$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 2 \\ 0 & -1 & -6 & -3 \\ 0 & 0 & 3p-13 & p+1 \end{array}\right]$$

$p = \frac{13}{3}$ lösn. saknas

$p \neq \frac{13}{3}$ entydig lösn

$z = \frac{p+1}{3p-13} \quad y = 3-6z = \frac{3(3p-13)-6(p+1)}{3p-13} = \frac{3p-45}{3p-13}$

$x = 2-y-3z = \frac{2(3p-13)-3p+45-3(p+1)}{3p-13} = \frac{16}{3p-13}$

5)

$$\begin{pmatrix}1&0&0&1\\1&1&0&-1\\1&1&1&-1\end{pmatrix}\overset{A}{\begin{bmatrix}1&1&1\\0&1&1\\0&0&1\\1&-1&-1\end{bmatrix}}\overset{u}{\begin{bmatrix}x\\y\\z\end{bmatrix}} = \begin{bmatrix}1&0&0&1\\1&1&0&-1\\1&1&1&-1\end{bmatrix}\overset{b}{\begin{bmatrix}6\\2\\3\\4\end{bmatrix}}$$

$$\begin{bmatrix}2&0&0\\0&3&3\\0&3&4\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}4\\-2\\1\end{bmatrix} \iff \begin{array}{l} x = 2\\ y = -\frac{11}{3}\\ z = 3\end{array}$$

medel fel: $Au - b = \begin{bmatrix}4/3\\-1/3\\0\\-11/3\end{bmatrix}$

$$\sqrt{\frac{1}{3}\left(\left(4/3\right)^2 + \left(-1/3\right)^2 + \left(11/3\right)^2\right)} = \sqrt{\frac{1}{3}\,\frac{138}{9}} = \sqrt{\frac{46}{9}} = \frac{\sqrt{46}}{3}$$

6)

$BXA = C \qquad X = B^{-1}CA^{-1}$

$$B^{-1} = \begin{bmatrix}-5&2\\-3&1\end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc}1&1&2&1&0&0\\3&2&-1&0&1&0\\0&1&1&0&0&1\end{array}\right] \sim \left[\begin{array}{ccc|ccc}1&1&2&1&0&0\\0&-1&-7&-3&1&0\\0&1&1&0&0&1\end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc}1&0&-5&-2&1&0\\0&-1&-7&-3&1&0\\0&0&-6&-3&1&1\end{array}\right] \sim \left[\begin{array}{ccc|ccc}1&0&0&1/2&1/6&-5/6\\0&-1&0&1/2&-1/6&-7/6\\0&0&-6&-3&1&1\end{array}\right]$$

$$A^{-1} = \frac{1}{6}\begin{bmatrix}3&1&-5\\-3&1&7\\3&-1&-1\end{bmatrix} \qquad X = \frac{1}{6}\begin{bmatrix}-5&2\\-3&1\end{bmatrix}\begin{bmatrix}2&1&-2\\0&1&2\end{bmatrix}\begin{bmatrix}3&1&-5\\-3&1&7\\3&-1&-1\end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix}-10&-3&14\\-6&-2&8\end{bmatrix}\begin{bmatrix}3&1&-5\\-3&1&7\\3&-1&-1\end{bmatrix} = \frac{1}{6}\begin{bmatrix}21&-27&15\\12&-16&8\end{bmatrix}$$

Eller $[B|C] \sim [E|D] \quad XA = D \iff [A^t|D^t] \sim [E|X^t]$

7a) $\vec{AB} = (-1,-1,1)$ $\quad \ell: (x,y,z) = (-1,1,4) + t(-1,-1,1)$

$\vec{OD} = \vec{OB} \pm 3\frac{\vec{AB}}{|\vec{AB}|} = (-1,1,4) \pm 3\frac{(-1,-1,1)}{\sqrt{3}}$

$= (-1,1,4) \pm \sqrt{3}(-1,-1,1)$ ligger 3 l.e. fr. B

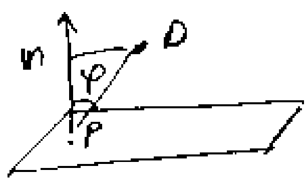
b) $\vec{CD} = (1,4,-6)$ $\quad \vec{AB}\cdot\vec{CD} = |\vec{AB}||\vec{CD}|\cos\varphi$

$\cos\varphi = \frac{-11}{\sqrt{3}\sqrt{53}}$ $\quad \varphi = \arccos\left(\frac{-11}{\sqrt{3}\sqrt{53}}\right)$

c) $\vec{AC} = (2,-5,2)$ $\quad n = \vec{AB}\times\vec{AC} = \begin{vmatrix} e_x & e_y & e_z \\ -1 & -1 & 1 \\ 2 & -5 & 2 \end{vmatrix}$

$n = (3,4,7)$ planets ekv: $3(x-0)+4(y-2)+7(z-3)=0$

$3x+4y+7z=29$

d) 

$d = |\vec{PD}|\cos\varphi = \frac{|\vec{PD}\cdot n|}{|n|}$

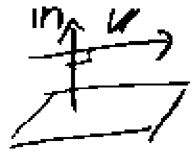
t.ex. $P = (1,-1,0) \Rightarrow \vec{PD} = (2,2,-1)$

$d = \frac{|(2,2,-1)\cdot(2,-3,4)|}{\sqrt{2^2+3^2+4^2}} = \frac{6}{\sqrt{29}}$

8a) $\ell: x=2t,\ y=t-1,\ z=3t+2$ sätt in i planet

$2t-2(t-1)+4(3t+2)=-5 \Leftrightarrow t=\frac{-15}{12}=-\frac{5}{4}$

$(-10/4,\ -9/4,\ -7/4)$

b) 

$v\cdot(1,-2,4)=0 \Leftrightarrow v_1-2v_2+4v_3=0$

t.ex. $v_1=0,\ v_2=2,\ v_3=1$ $\quad \ell:(x,y,z)=(1,-1,2)+t(0,2,1)$

c) 

$d = |\vec{P_0P}|\sin\varphi = \frac{|\vec{P_0P}\times v|}{|v|}$ $\quad P_0 = (0,-1,2)$

$v = (2,1,3)$

$d = \frac{|(1,0,0)\times(2,1,3)|}{\sqrt{14}} = \frac{\sqrt{10}}{\sqrt{14}} = \sqrt{5/7}$