3) $\left[\begin{array}{ccc|c} 1 & 2 & 2 & -1 \\ 2 & 3 & -p & -1 \\ -4 & p & -8 & 4 \end{array}\right] \overset{(-2)\ (4)}{\sim} \left[\begin{array}{ccc|c} 1 & 2 & 2 & -1 \\ 0 & -1 & -p-4 & 1 \\ 0 & p+8 & 0 & 0 \end{array}\right] \overset{(p+8)}{\sim} \left[\begin{array}{ccc|c} 1 & 2 & 2 & -1 \\ 0 & -1 & -p-4 & 1 \\ 0 & 0 & -(p+4)(p+8) & p+8 \end{array}\right]$

$p \neq -4$ och $p \neq -8$: $\begin{cases} x = -1 + 2/(p+4) = -(2+p)/(p+4) \\ y = 0 \\ z = -1/(p+4) \end{cases}$

$p = -8$: $\begin{cases} x = 1 - 10t \\ y = 4t - 1 \\ z = t \end{cases}$ $\quad p = -4$: lösn. saknas

4) $y = kx + m$ $\quad \underbrace{\begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} k \\ m \end{bmatrix}}_{u}$ minsta kvadratmet.: $A^T A u = A^T b$

$\begin{bmatrix} 1 & 2 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} u = \begin{bmatrix} 1 & 2 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 18 & 8 \\ 8 & 4 \end{bmatrix} u = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

$\left[\begin{array}{cc|c} 2 & 0 & 1 \\ 4 & 2 & 1 \end{array}\right]$ $\quad k = \frac{1}{2}$ $\quad m = \frac{1-4k}{2} = -\frac{1}{2}$ $\quad y = \frac{x-1}{2}$

5) $X = (2E - B)A^{-1}$ $\quad \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ -1 & 3 & 2 & 0 & 1 & 0 \\ 3 & 2 & 2 & 0 & 0 & 1 \end{array}\right] \overset{(1)\ (3)}{\sim} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 4 & 3 & 1 & 1 & 0 \\ 0 & -1 & -1 & -3 & 0 & 1 \end{array}\right]$

$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 0 & 1 \\ 0 & -1 & -1 & -3 & 0 & 1 \\ 0 & 0 & -1 & -11 & 1 & 4 \end{array}\right]$ $\quad X = \begin{bmatrix} 1 & -5 & -1 \\ 3 & 2 & -2 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ -8 & 1 & 3 \\ 11 & -1 & -4 \end{bmatrix} = \begin{bmatrix} 27 & -4 & -10 \\ -44 & 4 & 17 \\ 8 & -1 & -3 \end{bmatrix}$

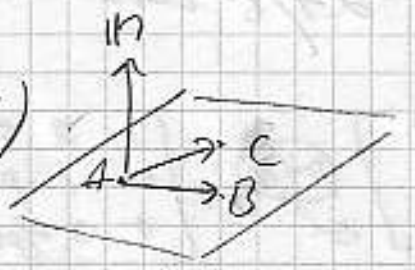
6a) $|u+2v|^2 = |u|^2 + 4u \cdot v + 4|v|^2$

$\cos\varphi = \frac{u \cdot v}{|u||v|} = \frac{25 - 4 - 36}{4 \cdot 2 \cdot 3} = \frac{-5}{8}$ $\quad \varphi = \arccos\left(\frac{-5}{8}\right)$

6b) $\begin{bmatrix} -1 & 3 & 2 \end{bmatrix} \begin{bmatrix} -1+2t \\ 3 \\ 2-5t \end{bmatrix} = 0 \Leftrightarrow 1 - 2t + 9 + 4 - 10t = 0$

$\Leftrightarrow t = \frac{7}{6}$

7) a)



$$n = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} e_x & e_y & e_z \\ -1 & 2 & 1 \\ 0 & 6 & 0 \end{vmatrix}$$

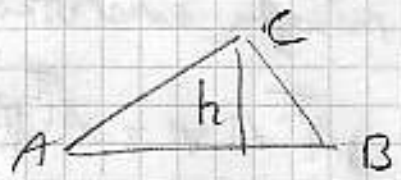
$$= (-6, 0, -6) = -6(1, 0, 1)$$

plan: $(1,0,1) \cdot (x-1, y, z-1) = 0 \Leftrightarrow$

$$x + z - 2 = 0$$

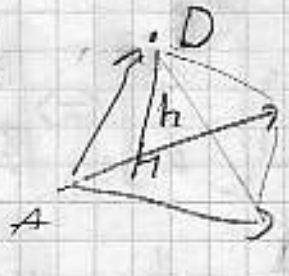
7b)

$$A = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2} \cdot 6 \cdot \sqrt{2} = 3\sqrt{2}$$



$$h = \frac{2A}{|\overrightarrow{AB}|} = \frac{6\sqrt{2}}{\sqrt{6}} = 2\sqrt{3}$$

7c)



$$V = \frac{1}{6}|\overrightarrow{AB} \times \overrightarrow{AC} \cdot \overrightarrow{AD}|$$

$$= \frac{1}{6}|-6(1,0,1) \cdot (-2,4,4)| = 2$$

$$h = \frac{3 \cdot \text{Vol}}{\text{Area}} = \frac{3 \cdot 2}{3\sqrt{2}} = \sqrt{2}$$

8a)

$$\left.\begin{aligned} -3 + 2t &= 2 + s \\ -1 + t &= 3 + 2s \\ 2 &= 4 + 2s \end{aligned}\right\} \quad \begin{aligned} -3 + 2t = 1 &\Leftrightarrow t = 2 \\ -1 + t = 1 &\Leftrightarrow t = 2 \end{aligned}$$

skärnpkt $(1; 1; 2)$

8b) $2(-3 + 2t) - (-1 + t) - 2 - 5 = 0 \Leftrightarrow 3t - 12 = 0$

skärnpkt $(5; 3; 2)$

8c)

$$v_1 = v - \frac{v \cdot n}{|n|^2} n = \begin{bmatrix} 2 \\ 2 \end{bmatrix} - \frac{\begin{bmatrix} 1 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}}{6} \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} = \frac{5}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

skärn: $2(2+s) - (3+2s) - (4+2s) - 5 = 0 \quad -2s - 8 = 0$

$$L: \begin{cases} x = -2 + t \\ y = -5 + t \\ z = -4 + t \end{cases}$$