SVAR Matte B/C B1/K1 10/3-2020

3

$$\begin{cases} -a+3b+c=0 \\ 2a+b-c=0 \end{cases} \xrightarrow{②+①} \begin{cases} -a+3b+c=0 \\ 7b+c=0 \end{cases}$$

$a=-4t$
$b=t$
$c=-7t$

4 a)

$$\begin{aligned} x+z &= x-2y \\ y+u &= x+3y \\ -2x+3z &= z-2u \\ -2y+3u &= z+3u \end{aligned}$$

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ -2 & 0 & 2 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 \end{bmatrix} \xrightarrow{②+①\cdot 2} \sim \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 4 & 2 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 \end{bmatrix} \begin{matrix} \leftarrow \\ \leftarrow \\ -①/2 \end{matrix}$$

$$\sim \left[\begin{array}{cccc} 1 & 2 & 0 & -1 \\ 0 & 2 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{cases} x = -2y+u = t+s \\ y = -t/2 \\ z = t \\ u = s \end{cases} \qquad \begin{bmatrix} t+s & -t/2 \\ t & s \end{bmatrix}$$

b) $y=kx+m$

$$x=\begin{bmatrix} k \\ m \end{bmatrix} \quad \begin{bmatrix} -1 & 1 \\ 1 & 1 \\ 3 & 1 \end{bmatrix} x = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 1 \\ 3 & 1 \end{bmatrix} x = \begin{bmatrix} -1 & 3 \\ \cdots & \end{bmatrix} \begin{bmatrix} -2 \\ 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 11 & 3 \\ 3 & 3 \end{bmatrix} x = \begin{bmatrix} 16 \\ 4 \end{bmatrix} \qquad 8k=12$$

$$k=\frac{3}{2} \qquad m=\frac{4}{3}-\frac{3}{2}=-\frac{1}{6}$$

$$\text{felvekt} = \begin{bmatrix} -2 \\ 2 \\ 4 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 1 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 3/2 \\ -1/6 \end{bmatrix} = \begin{bmatrix} -1/3 \\ 2/3 \\ -1/3 \end{bmatrix} \quad \text{medelfel} = \frac{1}{3\sqrt{3}}\sqrt{6} = \frac{\sqrt{2}}{3}$$

6a)

$$\left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{array}\right] \xrightarrow{-①+②} \sim \left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 0 & -4 & -1 & 1 \end{array}\right] \xrightarrow{\frac{3}{4}①+②} \sim \left[\begin{array}{cc|cc} 2 & 0 & 1/4 & 3/4 \\ 0 & -4 & -1 & 1 \end{array}\right]$$

$$A^{-1} = \frac{1}{8}\begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$$

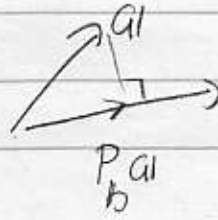
6b)

$$X = (A+2E)^{-1}B = -\frac{1}{2}\begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ -3 & 0 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 10 & 5 \\ -14 & -10 \end{bmatrix}$$

6a)

$$|3u-2v|^2 = 9|u|^2 - 12u\cdot v + 4|v|^2 = 36 + 12\cdot 2\cdot 3\cdot\frac{2}{3} + 36 = 120 \Rightarrow |3u-2v| = 2\sqrt{30}$$

6b)



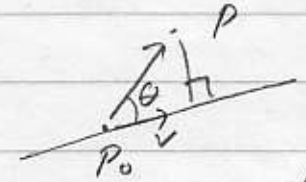
$$P_b a = \frac{a\cdot b}{|b|^2} b = \frac{-5}{25} b = -\frac{1}{5} b$$

7a) $L: \vec{r} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} + t\begin{bmatrix} -3 \\ 5 \end{bmatrix}$ $\perp$ t. ex. $\vec{r} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} + t\begin{bmatrix} 5 \\ 0 \\ -1 \end{bmatrix}$

b) $\vec{n} = \begin{vmatrix} e_x & e_y & e_z \\ 2 & 3 & -2 \\ 1 & -3 & 5 \end{vmatrix} = \begin{bmatrix} 9 \\ -12 \\ -9 \end{bmatrix}$ $\quad \vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$

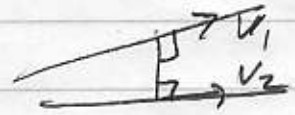
$3(x-1) - 4(y+1) - 3(z-2) = 0 \Leftrightarrow 3x - 4y - 3z - 1 = 0$

c) $3(2-3t) - (3+t) + 3 \cdot 2t = 5 \Leftrightarrow -4t = 2 \Rightarrow \left(\frac{7}{2}; \frac{5}{2}; -1\right)$

d) 

$d = |\overrightarrow{P_0P}| \sin\theta = \frac{|\overrightarrow{P_0P} \times \vec{v}|}{|\vec{v}|}$ $\quad \begin{vmatrix} e_x & e_y & e_z \\ -1 & 2 & 4 \\ 3 & 3 & 2 \end{vmatrix} = \begin{bmatrix} -8 \\ 14 \\ -9 \end{bmatrix}$

$d = \frac{\sqrt{8^2 + 14^2 + 9^2}}{\sqrt{22}}$



8a) $\vec{r}_1 = \begin{bmatrix} 1-2t \\ t-4 \\ t-1 \end{bmatrix}$ $\vec{r}_2 = \begin{bmatrix} 4+s \\ 2-s \\ 3+s \end{bmatrix}$ $(\vec{r}_1 - \vec{r}_2) \cdot \vec{v}_i = 0 \quad i = 1, 2$

$$\left.\begin{array}{l} (-3-2t-s)(-2) + (-6+t+s) \cdot 1 + (-4+t-s) \cdot 1 = 0 \\ (-3-2t-s) \cdot 1 + (-6+t+s) \cdot (-1) + (-4+t-s) \cdot 1 = 0 \end{array}\right\}$$

$$\Leftrightarrow \left.\begin{array}{l} 6t + 2s = 4 \\ -2t - 3s = 1 \end{array}\right\} \Leftrightarrow \left.\begin{array}{l} t = 1 \\ s = -1 \end{array}\right\} \quad \vec{r}_1 = \begin{bmatrix} -1 \\ -3 \\ 0 \end{bmatrix} \quad \vec{r}_2 = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$$

8b) $t - 2(1-2t) + 3(-1+3t) = 4 \Leftrightarrow 14t = 9 \Rightarrow \left(\frac{9}{14}; \frac{-4}{14}; \frac{13}{14}\right)$

8c) $\left.\begin{array}{l} 2x - 4y + z = 1 \\ 4x + 2y + 3z = 0 \end{array}\right\} \begin{matrix} (-2) \\ + \end{matrix} \Leftrightarrow \left\{\begin{array}{l} 2x - 4y + z = 1 \\ 10y + z = -2 \end{array}\right.$

$$\vec{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (4t + 2 + 10t + 1)/2 \\ t \\ -2 - 10t \end{bmatrix} = \begin{bmatrix} 3/2 \\ 0 \\ -2 \end{bmatrix} + t\begin{bmatrix} 7 \\ 1 \\ -10 \end{bmatrix}$$