Svar Matte B/C BI/KI 13/1-2011

(3)
$$\begin{bmatrix} 1 & 3 & 2 & 3 \\ 1 & a+1 & a & 2a-1 \\ 2 & a & 8 & 1 \end{bmatrix} \begin{matrix} \cdot(-1) \\ \\ \cdot(-2) \end{matrix} \sim \begin{bmatrix} 1 & 3 & 2 & 3 \\ 0 & a-2 & a-2 & 2a-4 \\ 0 & a-6 & 4 & -5 \end{bmatrix}$$

$a \neq 2$
$$\sim \begin{bmatrix} 1 & 3 & 2 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & a-6 & 4 & -5 \end{bmatrix} \begin{matrix} \\ \\ \cdot(-(a-6)) \end{matrix} \sim \begin{bmatrix} 1 & 3 & 2 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 10-a & 7-2a \end{bmatrix}$$

$a \neq 2, a \neq 10$: $x = 3 - 3y - 2z = 3 - \frac{39}{10-a} - \frac{14-4a}{10-a} = \frac{-23+a}{10-a}$

$y = 2 - z = 2 - \frac{7-2a}{10-a} = \frac{13}{10-a}$

$z = \frac{7-2a}{10-a}$

$a = 10$: $\begin{bmatrix} 1 & 3 & 2 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & -13 \end{bmatrix} \leftrightarrow$ $0 = -13$ Lösn. saknas

$a = 2$: $\begin{bmatrix} 1 & 3 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & -4 & 4 & -5 \end{bmatrix}$

$x = 3 - 3y - 2z = \frac{7}{4} - 5t$

$y = \frac{5}{4} + t$

$z = t$

(4)

$$1 = k\cdot 1 + m$$
$$1 = k\cdot 2 + m$$
$$2 = k\cdot 3 + m$$
$$3 = k\cdot 4 + m$$

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} k \\ m \end{bmatrix}}_{u} = \underbrace{\begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}}_{B}$$

$$A^t A u = A^t B$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{bmatrix} u = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 30 & 10 \\ 10 & 4 \end{bmatrix} u = \begin{bmatrix} 21 \\ 7 \end{bmatrix} \begin{matrix} \leftarrow \\ \boxed{-3} \end{matrix} \qquad \left.\begin{matrix} -2m = 0 \\ 10k + 4m = 7 \end{matrix}\right\}$$

$$m = 0 \quad k = 7/10 \quad \Rightarrow \quad y = \frac{7x}{10}$$

$$\text{modelfel} = \sqrt{\frac{|B - Au|^2}{4}} = \frac{1}{2}|B - Au|$$

$$= \frac{1}{2}\sqrt{\left(1 - \frac{7}{10}\right)^2 + \left(1 - \frac{14}{10}\right)^2 + \left(2 - \frac{21}{10}\right)^2 + \left(3 - \frac{28}{10}\right)^2}$$

$$= \frac{1}{20}\sqrt{30}$$

(5)

$$AX(E+B) = C$$

$$\underbrace{A^{-1}A}_{E} X \underbrace{(E+B)(E+B)^{-1}}_{E} = A^{-1}C(E+B)^{-1}$$

$$X = A^{-1}C(E+B)^{-1}$$

$$A^{-1}: \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right] \boxed{-1} \sim \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -1 & -1 & 1 \end{array}\right] \begin{matrix} \leftarrow \\ \boxed{2} \end{matrix} \sim \left[\begin{array}{cc|cc} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \end{array}\right]$$

$(E+B)^{-1}$:
$$\begin{bmatrix} 2 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \overset{\cdot(-2)}{\sim} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & -2 & 1 & -2 \end{bmatrix} \overset{\cdot(-1/2)}{\sim}$$
$$\sim \begin{bmatrix} 1 & 0 & 1/2 & 0 \\ 0 & 1 & -1/2 & 1 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 4 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$
$$= \frac{1}{2}\begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$$

(6) (a) $|u-v|^2 = (u-v)\cdot(u-v) = u\cdot u - 2u\cdot v + v\cdot v = |u|^2 - 2u\cdot v + |v|^2 = 4+4+3 = 11$

(b) $u = tb \quad (a-tb)\cdot b = 0$

$$t = \frac{a\cdot b}{b\cdot b} = \frac{a\cdot b}{|b|^2} = \frac{1}{2} \Rightarrow u = \frac{1}{2}\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

(7) (a) $n = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} e_x & e_y & e_z \\ -1 & 2 & 1 \\ 0 & 6 & 0 \end{vmatrix}$

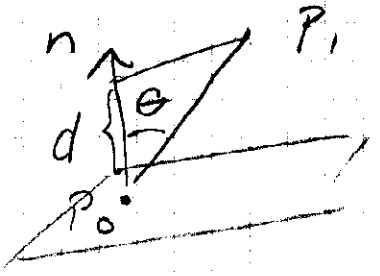
$= (-6, 0, -6) \parallel (1, 0, 1) \qquad \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x-1 \\ y-0 \\ z-1 \end{bmatrix} = 0$

$x + z - 2 = 0$

(b) $|\overrightarrow{AB}| = \sqrt{6} \qquad \cos\theta = \dfrac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|} = \dfrac{\begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}}{\sqrt{6}\ \sqrt{18}}$

$\theta = \arccos\dfrac{-1}{\sqrt{3}}$

(c)



$$d = |\overrightarrow{P_0P_1}| \cos\theta = \frac{|\overrightarrow{P_0P_1} \cdot n|}{|n|}$$

t. ex. $P_0 = (0,0,3)$ $P_1 = (0,0,5)$

$$\Rightarrow d = \frac{\left|\begin{bmatrix}0\\0\\2\end{bmatrix} \cdot \begin{bmatrix}1\\-2\\1\end{bmatrix}\right|}{\sqrt{6}} = \frac{2}{\sqrt{6}}$$

(8) (a)

$$\left.\begin{aligned} 2-2t &= 1+s \\ 2+t &= 1-s \\ 1-2t &= 3+2s \end{aligned}\right\} \Rightarrow 4-t=2 \quad t=2,\ s=-3$$

$1-2\cdot 2 = 3+2(-3)$ ok

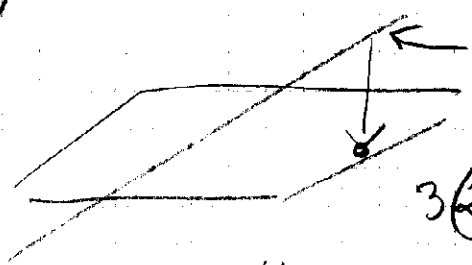
skæringspkt $(-2,4,-3)$

(b)

$$3(2-2t)-4(1-2t) = -2 \iff t = \frac{-4}{2} = -2$$

$$\Rightarrow (6,0,5)$$

(c)



t. ex. $\begin{bmatrix}2\\2\\1\end{bmatrix} + t\begin{bmatrix}3\\0\\-4\end{bmatrix}$

$$3(2+3t) - 4(1-4t) + 2 = 0$$

$$t = \frac{-4}{25}$$

$$\Rightarrow \text{proj}: \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}6\\0\\5\end{bmatrix} + s\left(\begin{bmatrix}2\\2\\1\end{bmatrix} - \frac{4}{25}\begin{bmatrix}3\\0\\-4\end{bmatrix} - \begin{bmatrix}6\\0\\5\end{bmatrix}\right)$$

$$= \begin{bmatrix}6\\0\\5\end{bmatrix} + s\begin{bmatrix}-114/25\\2\\-84/25\end{bmatrix}$$