Matte del B/c BI/KI 14/5-2011

(3)
$$\left[\begin{array}{ccc|c} 1 & 2 & a & 1 \\ 1 & 3 & 2 & 2 \\ 2 & a & 8 & 2 \end{array}\right] \begin{matrix} \cdot(-1) \\ \cdot(-2) \\ \end{matrix} \sim \left[\begin{array}{ccc|c} 1 & 2 & a & 1 \\ 0 & 1 & 2-a & 1 \\ 0 & a-4 & 8-2a & 0 \end{array}\right] \cdot(a-4)$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & a & 1 \\ 0 & 1 & 2-a & 1 \\ 0 & 0 & (a-4)^2 & -(a-4) \end{array}\right] \quad a=4 \Rightarrow \begin{cases} x = 1-2y-4t = -1-8t \\ y = 1+2t \\ z = t \end{cases}$$

$$a\neq 4: \begin{cases} x = 1-2y-az = 1+\frac{4}{a-4}+\frac{a}{a-4} = \frac{2a}{a-4} \\ y = 1+\frac{2-a}{a-4} = -\frac{2}{a-4} \\ z = -\frac{1}{a-4} \end{cases}$$

(4)
$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 3 \\ 3 \end{bmatrix} \Leftrightarrow \left[\begin{array}{cc|c} 30 & 10 & 24 \\ 10 & 4 & 8 \end{array}\right]$$

$-2y = 0 \quad x = 4/5 \quad \vec{fel} = [-1/5 \;\; 3/5 \;\; -3/5 \;\; 1/5]^T$

$\text{modul } fel = \frac{1}{5}\cdot\frac{1}{\sqrt{4}}\sqrt{20} = 1/\sqrt{5}$

(5)
$$X = -A^{-1}B(E-B)^{-1} = -\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ -1 & 1 \end{bmatrix}^{-1}$$

$$= -\begin{bmatrix} 3 & 10 \\ -1 & -4 \end{bmatrix}\left(-\frac{1}{2}\right)\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 13 & 6 \\ -5 & -2 \end{bmatrix}$$

(6a)
$$17 = |u-v|^2 = |u|^2 - 2u\cdot v + |v|^2 \Leftrightarrow |v|^2 + 2|v| - 8 = 0$$

$$|v| = -1 \pm \sqrt{1+8} = -1 \pm 3 \quad |v| = 2$$

(6b)
$$u = tb \quad (a-tb)\cdot(a+b) = 0$$

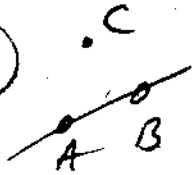
$$t = \frac{a\cdot(a+b)}{b\cdot(a+b)} = \frac{5+5}{5+11} = \frac{10}{16} = \frac{5}{8}$$

(7a) $n = \begin{vmatrix} e_x & e_y & e_z \\ 2 & 1 & 0 \\ -5 & -2 & 1 \end{vmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

$1\cdot(x-3) - 2(y-2) + 1\cdot(z-1) = 0 \Leftrightarrow x - 2y + z = 0$

(7b) $\left|\frac{1}{2}(\overrightarrow{AB}+\overrightarrow{AC})\right| = \frac{1}{2}\left|[-3\ -1\ 1]^T\right| = \frac{1}{2}\sqrt{11}$

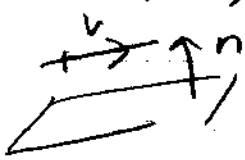
$\cos\theta = \frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{|\overrightarrow{AB}||\overrightarrow{AC}|} = \frac{-12}{\sqrt{5}\sqrt{30}}$ $\theta = \arccos\left(\frac{-2\sqrt{6}}{5}\right)$

(7c) 

$d = \frac{|\overrightarrow{AB}\times\overrightarrow{AC}|}{|\overrightarrow{AB}|} = \frac{|[1\ -2\ 1]^T|}{|[2\ 1\ 0]^T|} = \frac{\sqrt{6}}{\sqrt{5}}$

(8a) $2(5-3t) - (3-2t) + 4(1+2t) + 1 = 0 \Leftrightarrow t = \frac{12}{-4}$

$\Rightarrow (11; 9; -7)$

(8b) 

$v\cdot n = 0 \quad [-3\ -2\ 2]\cdot[a\ b\ c]^T = 0$

e.ex. $n = [0\ 1\ 1]^T$

$\Rightarrow 0\cdot(x+1) + 1\cdot(y-2) + 1\cdot(z+3) = 0 \Leftrightarrow y + z + 1 = 0$

(8c) 

$v_\perp = u - \frac{u\cdot v}{|v|^2}v = \begin{bmatrix} -6 \\ -1 \\ -4 \end{bmatrix} - \frac{[-6\ -1\ -4][-3\ -2\ 2]^T}{17}\begin{bmatrix} -3 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} -6 \\ -1 \\ -4 \end{bmatrix} - \frac{12}{17}\begin{bmatrix} -3 \\ -2 \\ 2 \end{bmatrix}$

$L_1 : \overrightarrow{OP} = \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} + t\,v_\perp$