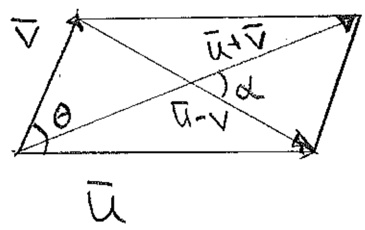
ÖB PMI

Ex.1 $\|\bar{u}\| = 4$ $\qquad\alpha$ sökes!



Parallellogramens omkrets

$$P = 2\|\bar{u}\| + 2\|\bar{v}\| = 12$$

$$\Leftrightarrow \|\bar{v}\| = 2$$

Vi antar att vinkeln mellan $\bar{u}$ och $\bar{v}$ är θ

Vi vet att parallellogramens area är $6\,cm^2$.

$$\Rightarrow T = \|\bar{u} \times \bar{v}\| = \underbrace{\|\bar{u}\|}_{=4}\ \underbrace{\|\bar{v}\|}_{=2} \sin\theta = 6$$

$$\Rightarrow \sin\theta = \frac{6}{8} = \frac{3}{4}$$

Med figurens beteckningar blir diagonalerna $\bar{u}+\bar{v}$ och $\bar{u}-\bar{v}$ och vi får att

$$\pm(\bar{u}+\bar{v})\cdot(\bar{u}-\bar{v}) = \|\bar{u}+\bar{v}\|\,\|\bar{u}-\bar{v}\|\cos\alpha$$

$$\Rightarrow \alpha = \arccos \pm\frac{(\bar{u}+\bar{v})\cdot(\bar{u}-\bar{v})}{\|\bar{u}+\bar{v}\|\,\|\bar{u}-\bar{v}\|}.$$

Vi behöver veta $(\bar{u}+\bar{v})\cdot(\bar{u}-\bar{v})$, $\|\bar{u}+\bar{v}\|$ och $\|\bar{u}-\bar{v}\|$

$$(\bar{u}+\bar{v})\cdot(\bar{u}-\bar{v}) = \|\bar{u}\|^2 \underbrace{-\bar{u}\cdot\bar{v} + \bar{v}\cdot\bar{u}}_{=0} - \|\bar{v}\|^2 = 16 - 4 = 12$$

$$\|\bar{u}+\bar{v}\|^2 = (\bar{u}+\bar{v})\cdot(\bar{u}+\bar{v}) = \|\bar{u}\|^2 + 2\bar{u}\cdot\bar{v} + \|\bar{v}\|^2$$

$$= \|\bar{u}\|^2 + \|\bar{v}\|^2 + 2\|\bar{u}\|\,\|\bar{v}\|\cos\theta =$$

$$= 16 + 4 + 2\cdot 4\cdot 2\cos\theta =$$

$$= 20 + 16\cos\theta$$

$$= 4(5 + 4\cos\theta)$$

$$\|\bar{u}-\bar{v}\|^2 = (\bar{u}-\bar{v})\cdot(\bar{u}-\bar{v}) = \|\bar{u}\|^2 - 2\bar{u}\cdot\bar{v} + \|\bar{v}\|^2$$

$$= \|\bar{u}\|^2 + \|\bar{v}\|^2 - 2\|\bar{u}\|\,\|\bar{v}\|\cos\theta$$

$$= 4(5-4\cos\theta)$$

Vi har då att

$$\|\bar{u}+\bar{v}\|\,|\bar{u}-\bar{v}| = \sqrt{4(5+4\cos\theta)4(5-\cos\theta)} =$$

$$= 4\sqrt{25-16\cos^2\theta} = 4\sqrt{25-16(1-\sin^2\theta)} =$$

$$= \left\{ \text{Vet sedan tidigare att } \sin\theta = \frac{3}{4} \right\} =$$

$$= 4\sqrt{25-16\left(1-\frac{9}{16}\right)} = 4\sqrt{25-16+9} = 4\sqrt{18} = \underline{\underline{12\sqrt{2}}}$$

Nu kan vi beräkna α

$$\alpha = \arccos\frac{12}{12\sqrt{2}} = \arccos\frac{1}{\sqrt{2}} = \underline{\underline{\frac{\pi}{4}}}$$