G.

PM5

Ex.1 Jacobis metod

$$[A \quad I] = \begin{bmatrix} 2 & 3 & -1 & 1 & 0 & 0 \\ 6 & 8 & -2 & 0 & 1 & 0 \\ 2 & 5 & -2 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3/2 & -1/2 & 1/2 & 0 & 0 \\ 0 & -1 & 1 & -3 & 1 & 0 \\ 0 & 2 & -1 & -1 & 0 & 1 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 0 & 1 & 4 & 3/2 & 0 \\ 0 & 1 & -1 & 3 & -1 & 0 \\ 0 & 0 & 1 & -7 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 3 & -1/2 & -1 \\ 0 & 1 & 0 & -4 & 1 & 1 \\ 0 & 0 & 1 & -7 & 2 & 1 \end{bmatrix} = [I \quad A^{-1}]$$

Ex.2

$$BX + X = A^T \iff BX + IX = A^T \iff$$

$$(B+I)X = A^T \iff X = (B+I)^{-1} A^T \quad \left(\text{om } (B+I)^{-1} \text{ existerar.} \right)$$

$$B + I = \begin{bmatrix} 0 & 2 \\ 2 & -2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

$$\det(B+I) = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -1 - 4 = -5 \neq 0$$

$(B+I)^{-1}$ existerar, $(B+I)^{-1} = \frac{1}{-5}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}$

$$\Rightarrow X = -\frac{1}{5}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}^T =$$

$$= \frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 9 & 12 & 15 \\ -2 & -1 & 0 \end{bmatrix}$$

Ex. 3

$$(2A+X)B^{-1} = I \iff (2A+B)\underbrace{B^{-1}B}_{=I} = IB$$

$$\iff 2A + X = IB \iff X = IB - 2A$$

$$X = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -2 & -1 \end{bmatrix}$$

Ex. 4

Villkor: $AB = BA$

$$AB = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & x & 4 \\ 2 & 2 & x \end{bmatrix} = \begin{bmatrix} 6 & x+6 & x+8 \\ 5 & 6 & 2x+2 \\ 3 & x+2 & 6 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 2 & 2 \\ 2 & x & 4 \\ 2 & 2 & x \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 5 \\ x+8 & 6 & 2x+2 \\ x+6 & x+2 & 6 \end{bmatrix}$$

Identifiering ger

$$x + 8 = 5 \quad \wedge \quad x + 6 = 3$$

$$\iff$$

$$\underline{\underline{x = -3}}$$

PM5

ÖB

EX.1

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$$

Sätt: $X = \begin{bmatrix} x_1 & x_3 \\ x_2 & x_4 \end{bmatrix}$ (2 x 2 matris)

- Villkor: $AX = A$

- $$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} x_1 & x_3 \\ x_2 & x_4 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 & x_3 + 2x_4 \\ 3x_1 + 6x_2 & 3x_3 + 6x_4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$$

Vilket ger

$x_1 + 2x_2 = 1$

$x_3 + 2x_4 = 2$

$x_1 + 2x_2 = 1$

$x_3 + 2x_4 = 2$

På matrisform $\begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 & 2 \\ 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 & 2 \end{bmatrix} \sim$

- $\begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ x_1, x_3 bundna

x_2, x_4 fri

$$\Rightarrow \begin{cases} x_1 = 1 - 2s \\ x_2 = s \\ x_3 = 2 - t \\ x_4 = t \end{cases}$$

Svar: $X = \begin{bmatrix} 1-2s & 2-2t \\ s & t \end{bmatrix}$

$s, t \in \mathbb{R}$