Matematik, T3.
LMA 164 B
18 december 2013

1. **a.** $\lim\limits_{x\to\infty} \dfrac{x^3+2x^2+5}{x+3x^2+2x^3} = \lim\limits_{x\to\infty} \dfrac{1+\frac{2}{x}+\frac{5}{x^3}}{\frac{1}{x^2}+\frac{3}{x}+2} = \underline{\underline{\dfrac{1}{2}}}.$

b. $\lim\limits_{x\to 2} \dfrac{x^2+x-6}{\sqrt{4x+1}-\sqrt{2x+5}} = \lim\limits_{x\to 2} \dfrac{(x-2)(x+3)\left[\sqrt{4x+1}+\sqrt{2x+5}\right]}{4x+1-(2x+5)}$

$= \lim\limits_{x\to 2} \dfrac{1}{2}(x+3)\left[\sqrt{4x+1}+\sqrt{2x+5}\right] = \dfrac{1}{2}\cdot 5\,(3+3) = \underline{\underline{15}}.$

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2. **a.** $\cos 3x = \frac{1}{2}$ Allmänt: $3x = \pm 60^\circ + n\cdot 360^\circ$

$x = \pm 20^\circ + n\cdot 120^\circ$ Om $0^\circ \le x \le 360^\circ$: $\underline{20^\circ, 100^\circ, 140^\circ, 220^\circ, 260^\circ, 340^\circ}$

b. $\sqrt{3}\tan x - 2\sin x = 0$

$\sin x\left[\dfrac{\sqrt{3}}{\cos x} - 2\right] = 0$ dvs $\sin x = 0$ eller $\cos x = \dfrac{\sqrt{3}}{2}$

Med $0^\circ \le x \le 180^\circ$: $\underline{0^\circ, 30^\circ, 180^\circ}.$

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3. **a.** $\ln(x-3) - \ln(x-1) = \ln(x-6) - \ln(x-5)$ $(\because\ x>6)$

$\ln\left[(x-3)\cdot(x-5)\right] = \ln\left[(x-1)\cdot(x-6)\right]$

$x^2-8x+15 = x^2-7x+6$ $\qquad \underline{x=9}$ (ok)

b. $3\cdot 5^{x+2} = 13 - 2\cdot 5^{x+3}$ $\qquad$ Sätt $t = 5^{x+2}$

$3\cdot t = 13 - 2\cdot t\cdot 5$, $13t = 13$, $t=1$ $\qquad \underline{x=-2}$

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4. $f(x) = x^3 - 3x^2 - 24x + 28$; $D_f = [-3, 5]$

$f'(x) = 3x^2 - 6x - 24 = 3(x^2-2x-8) = 3(x+2)(x-4)$

x	-3		-2		4		5
$x+2$		$-$	0	$+$		$+$	
$x-4$		$-$		$-$	0	$+$	
$f'(x)$		$+$	0	$-$	0	$+$	
$f(x)$	46	$\nearrow$	56	$\searrow$	-52	$\nearrow$	-42

Lok. max.: $(-2, 56)$, $(5, -42)$

Lok. min.: $(-3, 46)$, $(4, -52)$

Största värde: 56

Minsta värde: -52.

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5. $z^4 = -8 + 8\sqrt{3}\,j \;(= 8a)$

$z = re^{j\phi}$, $z^4 = r^4 e^{j4\phi}$

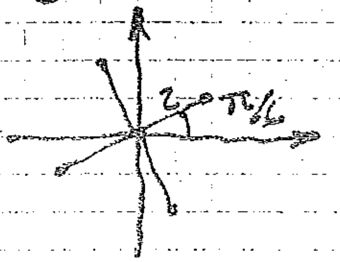
$-8 + 8\sqrt{3}\,j = 8(-1+\sqrt{3}\,j) =$

$= 8 \cdot 2 \cdot e^{j\frac{2\pi}{3}}$

Bel. lika: $r^4 = 16$ $\qquad r = 2$

Arg. "lika": $4\phi = \frac{2\pi}{3} + k \cdot 2\pi \qquad \phi = \frac{\pi}{6} + k \cdot \frac{\pi}{2}$

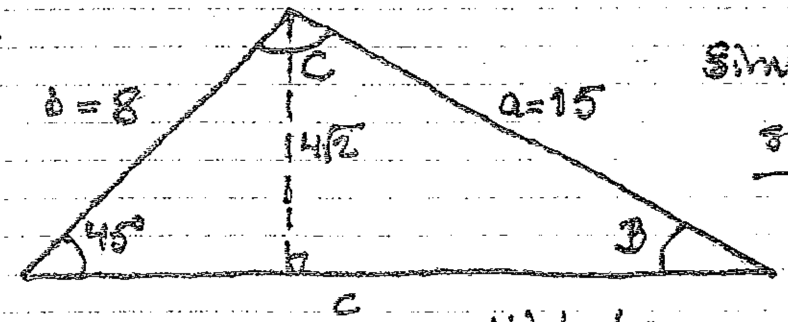
$k = 0, 1, 2, 3$



$z_1 = \underline{\sqrt{3} + j}$, $z_2 = \underline{-1 + j\sqrt{3}}$

$z_3 = -z_1 = \underline{-\sqrt{3} - j}$, $z_4 = -z_2 = \underline{1 - j\sqrt{3}}$.

6.



Sinussatsen:

$$\frac{\sin B}{8} = \frac{\sin 45}{15}$$

ger $B \approx \underline{22.2^\circ}$

Vinkelsumman 180 ger $C \approx \underline{112.8^\circ}$

och $\frac{\sin C}{c} = \frac{\sin 45}{15}$ ger $c \approx \underline{19.5}$.

b. Av fig. ovan inses: $a < 4\sqrt{2}$ ger ingen lösn.

$a = 4\sqrt{2}$ ger en lösn.

$4\sqrt{2} < a < 8$ ger två lösn.

$a \geq 8$ ger en lösn.

7.

$$f(x) = \begin{cases} 3 - x^2 & \text{om } x < 1 \quad f_1(x) \\ x^2 + ax + b & \text{om } 1 \leq x \leq 4 \quad f_2(x) \\ 3x - 7 & \text{om } x > 4 \quad f_3(x) \end{cases}$$

a. Krav för kontinuitet:

$$\begin{cases} f_1(1) = f_2(1) \\ f_2(4) = f_3(4) \end{cases} \quad \begin{cases} 2 = 1 + a + b \\ 16 + 4a + b = 5 \end{cases} \quad \begin{cases} a + b = 1 \quad (-1) \\ 4a + b = -11 \end{cases} \quad 3a = -12$$

$\underline{a = -4}$, $\underline{b = 5}$

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(7.) b. Är f der.-bar?

Krav: $\begin{cases} f_1'(1) = f_2'(1) \\ f_2'(4) = f_3'(4) \end{cases}$

$f_1'(x) = -2x$, $f_2'(x) = 2x + \overset{-4}{a}$, $f_3'(x) = 3$

$\begin{cases} -2 = 2-4 & \text{ok} \\ 8-4 = 3 & \text{Nej!} \end{cases}$

f blir ej en der.-bar fkn. ($f_-'(4) = 4$, $f_+'(4) = 3$)

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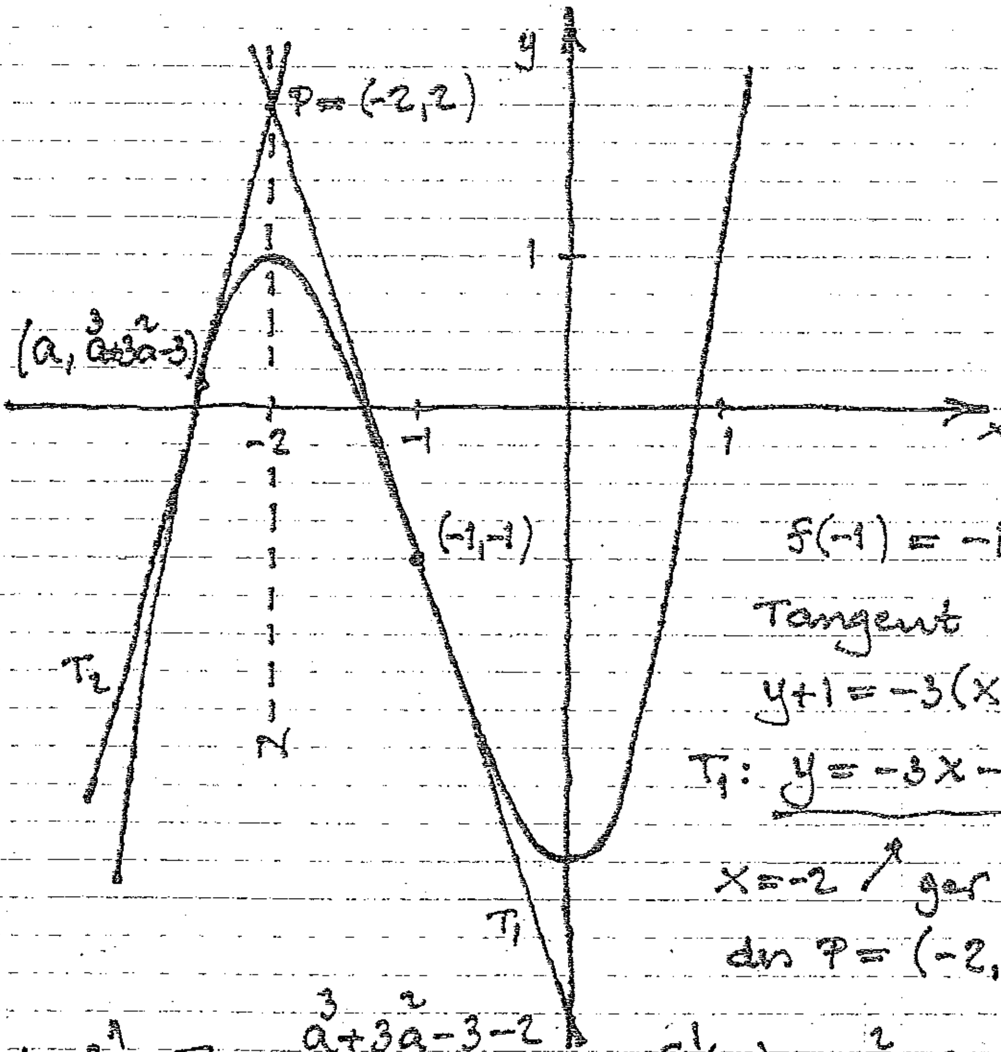
8. $f(x) = x^3 + 3x^2 - 3$

$f'(x) = 3x^2 + 6x = 3x(x+2)$



$f(-1) = -1$, $f'(-1) = -3$

Tangent i $(-1,-1)$:

$y+1 = -3(x+1)$

T_1: $\underline{y = -3x - 4}$

$x = -2$ ger $y = 2$

dvs $P = (-2, 2)$

Lutn. för T_2: $\dfrac{a^3+3a^2-3-2}{a+2} = f'(a) = 3a^2 + 6a$

$a^3 + 3a^2 - 5 = 3a^3 + 12a^2 + 12a$, $\quad 2a^3 + 9a^2 + 12a + 5 = 0$

$a_1 = -1$ (förstås!): $(a+1)(2a^2 + 7a + 5) = 0 \qquad a_2 = -1$

$(a+1)(a+1)(2a+5) = 0 \qquad \underline{a_3 = -\frac{5}{2}}$ ger vår sökta punkt $\left(-\frac{5}{2}, \frac{1}{8}\right)$

En normal genom P: $\underline{x = -2}$ (OBS! $f'(-2) = 0$)