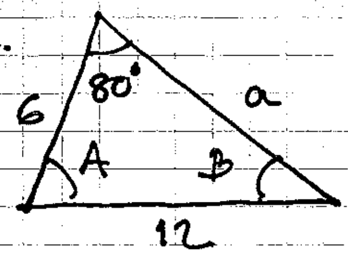
Mat. T.B. LMA 164 B
10 februari 2014

1. a.



Anv. sinus-satsen:

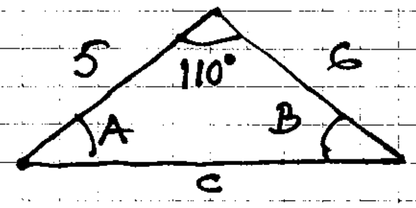
$$\frac{\sin B}{6} = \frac{\sin 80}{12} \text{ ger } \sin B \approx 0.492404 \quad \underline{B \approx 29.5}$$

(ty 150.5 orimligt)

Sedan: $A = 180 - B - 80 \approx \underline{70.5}$.

$$\frac{\sin A}{a} = \frac{\sin 80}{12} \text{ ger } \underline{a \approx 11.5}.$$

b.



Anv. cos-satsen:

$$c^2 = 25 + 36 - 60 \cdot \cos 110 \quad \underline{c \approx 9.0}$$

$$\frac{\sin A}{6} = \frac{\sin 110}{c} \text{ ger } \underline{A \approx 38.6}$$

$$B = 180 - A - 110 \text{ ger } \underline{B \approx 31.4}.$$

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2. a.

$$\lim_{x\to\infty} \frac{\sqrt{x} + x^2 + 2}{1 + 3x + 4x^2} = \lim_{x\to\infty} \frac{\frac{1}{x^{3/2}} + 1 + \frac{2}{x^2}}{\frac{1}{x^2} + \frac{3}{x} + 4} = \underline{\frac{1}{4}}.$$

b.

$$\lim_{x\to 3} \frac{x^2 - x - 6}{\sqrt{5x+1} - \sqrt{3x+7}} = \left(\text{"}\frac{0}{0}\text{"}\right) =$$

$$= \lim_{x\to 3} \frac{(x^2 - x - 6)(\sqrt{5x+1} + \sqrt{3x+7})}{5x+1 - (3x+7)} =$$

$$= \lim_{x\to 3} \frac{(x+2)(x-3)(\sqrt{5x+1} + \sqrt{3x+7})}{2(x-3)} = \frac{5(4+4)}{2} = \underline{20}.$$

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3. a. $2 \cdot 3^{x+4} = 10 - 12 \cdot 3^{x+2}$ Sätt $t = 3^{x+2}$

$2 \cdot 9 \cdot t = 10 - 12 \cdot t$, $30t = 10$, $t = \frac{1}{3}$ $\underline{x = -3}$.

b. $\ln(x-4) - \ln(x-6) = \ln(x-2) - \ln(x-5)$ $[x > 6]$

$(x-4)(x-5) = (x-2)(x-6)$

$x^2 - 9x + 20 = x^2 - 8x + 12$ $\underline{x = 8}$ (OK)

4. a. $\tan 6x = \frac{1}{\sqrt{3}}, \ 0 < x < 90$

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$6x = 30 + n \cdot 180$

$x = 5 + n \cdot 30, \ n = 0, 1, 2$ $\qquad \underline{x = 5 \text{ el. } 35 \text{ el. } 65}.$

b. $4\cos x + 3\sin x = \frac{5\sqrt{3}}{2}, \quad 0 < x < 90$

V.L. $= c \cdot \sin(x + \alpha)$ där $c = 5$, $\quad \sin\alpha = \frac{4}{5}$, $\cos\alpha = \frac{3}{5}$

$\sin(x+\alpha) = \frac{\sqrt{3}}{2}, \quad x + \alpha = \begin{cases} 60 + n \cdot 360 \\ 120 + n \cdot 360 \end{cases}$

$\alpha \approx 53.1$

$\alpha < 60$ ger $x = 60 - \alpha \approx \underline{6.9}$. eller $120 - \alpha \approx \underline{66.9}$

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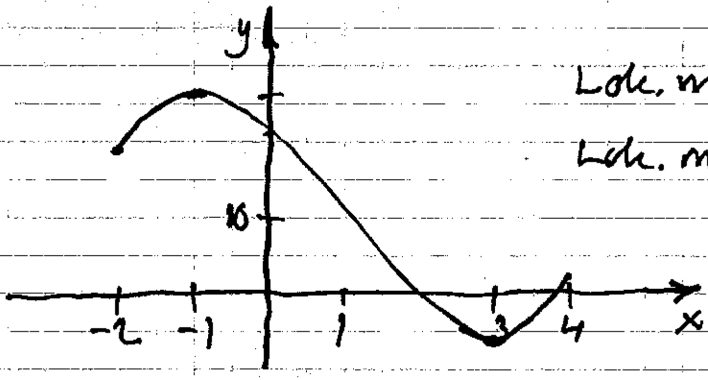
5. $f(x) = x^3 - 3x^2 - 9x + 21 \ ; \ x \in [-2, 4]$.

$f'(x) = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3) = 3(x+1)(x-3)$

	-2		-1		0		3		4
f'		+	0	−		−	0	+	
f		↗		↘				↗	

$f(-2) = 19, \ f(-1) = 26$

$f(3) = -6, \ f(4) = 1$



Lok. min. i $(-2, 19)$ och $(3, -6)$

Lok. max. i $(-1, 26)$ och $(4, 1)$

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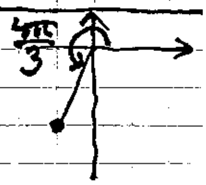
6. $z^4 = -\frac{1}{32} - \frac{\sqrt{3}}{32} j = \frac{1}{16}\left(\frac{-1}{2} - \frac{\sqrt{3}}{2} j\right)$



$|H.L.| = \frac{1}{16} \quad \therefore |z| = \frac{1}{2}$

Med $z = re^{j\theta}$, $z^4 = r^4 e^{4j\theta}$ får vi $r^4 = \frac{1}{16}$, $r = \frac{1}{2}$

och $4\theta = \frac{4\pi}{3} + 2k\pi$, $\theta = \frac{\pi}{3} + k \cdot \frac{\pi}{2}$; $k = 0, 1, 2, 3$

$k = 0$ ger $z_1 = \frac{1}{2}\left(\cos\frac{\pi}{3} + j \sin\frac{\pi}{3}\right) = \frac{1}{4}(1 + \sqrt{3} j)$ osv :

$k = 1$ ger $z_2 = \frac{1}{4}(-\sqrt{3} + j)$, $z_3 = \frac{1}{4}(-1 - \sqrt{3} j)$, $z_4 = \frac{1}{4}(\sqrt{3} - j)$

7. $f(x) = \begin{cases} x^2+6x+6 & \text{då } x<-1 \\ a+bx-x^2 & \text{då } -1\le x\le 2 \\ x^2-6x+12 & \text{då } x>2 \end{cases}$

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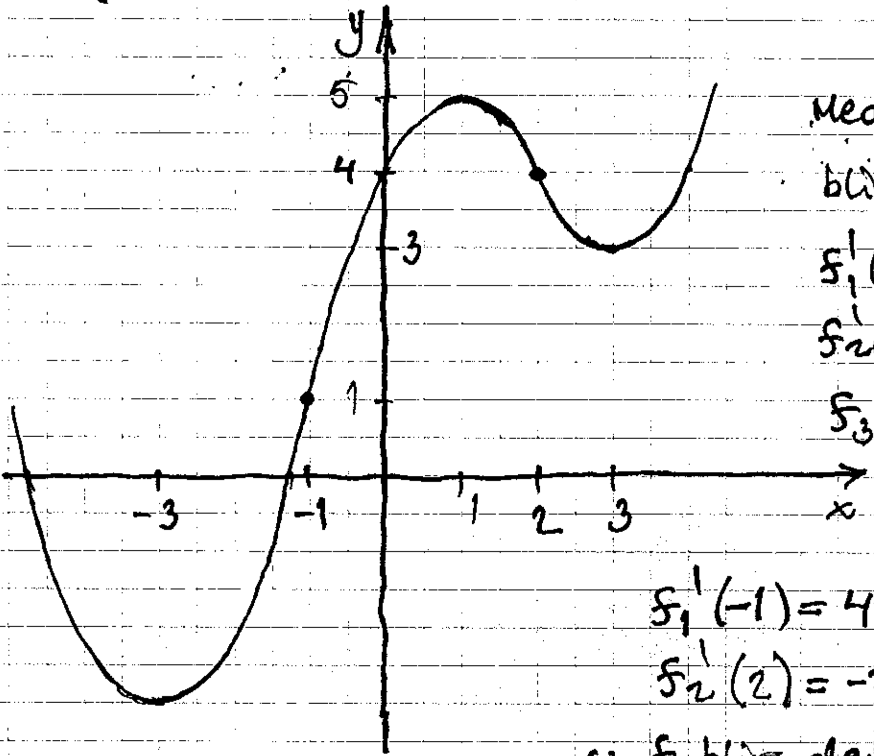
Kalla H.L. för f_1, f_2, f_3.

Krav för kont.: $f_1(-1)=f_2(-1)$, $f_2(2)=f_3(2)$

dvs $\begin{cases} 1 = a-b-1 \\ a+2b-4=4 \end{cases}$ $\begin{cases} a-b=2 \\ a+2b=8 \end{cases}$ (-1) $\begin{cases} a-b=2 \\ 3b=6 \end{cases}$ $\begin{cases} a=4 \\ b=2 \end{cases}$



Med dessa värden: blir f deriverbar?

$f_1'(x)=2x+6$

$f_2'(x)=2-2x$

$f_3'(x)=2x-6$

$f_1'(-1)=4=f_2'(-1)$

$f_2'(2)=-2=f_3'(2)$

$\therefore$ f blir deriverbar.

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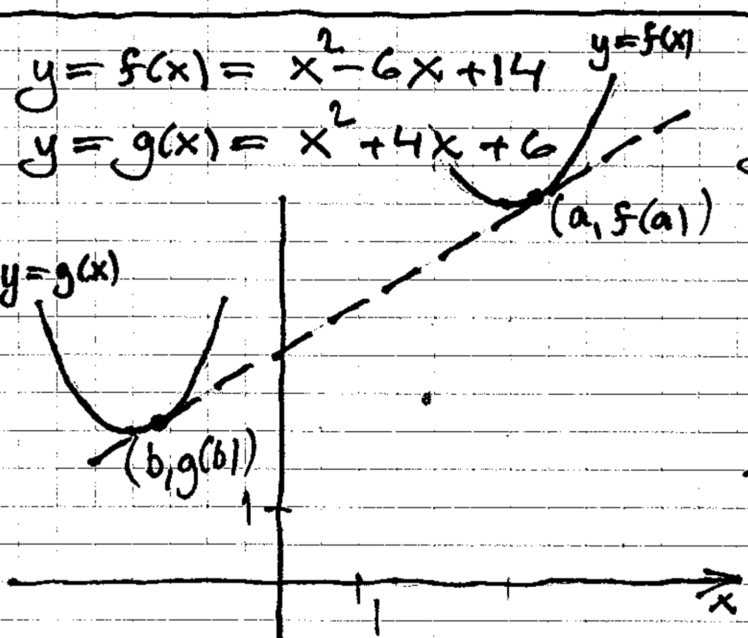
8. $y=f(x)=x^2-6x+14$

$y=g(x)=x^2+4x+6$



$f'(x)=2x-6$

$g'(x)=2x+4$

$k_t = f'(a)=g'(b)=\dfrac{f(a)-g(b)}{a-b}$

$2a-6=2b+4=$

$=\dfrac{a^2-6a+14-b^2-4b-6}{a-b}$

Första likheten: $a=b+5$

$\therefore 2b+4=\frac{1}{5}(b^2+10b+25-6b-30+14-b^2-4b-6)$

$10b+20=3$ $\quad b=\dfrac{-17}{10}$, $a=\dfrac{33}{10}$ (Jfr. fig.!)