Lösn. till Basåret matematik D. LMA164. 2013-03-15

1. a) $f(x) = \frac{9}{4x^2}$. $\frac{1}{h}\left[\frac{9}{4(x+h)^2} - \frac{9}{4x^2}\right] = \frac{9}{4h}\left\{\frac{\cancel{x^2} - \cancel{x^2} - 2xh - h^2}{x^2(x+h)^2}\right\}$

$= \frac{9}{4}\left\{\frac{-2x-h}{x^2(x+h)^2}\right\} \xrightarrow[\text{då } h\to 0]{} \frac{9}{4}\cdot\left[\frac{-2x}{x^2\cdot x^2}\right] = -\frac{9}{2x^3}$

b) $f(3) = \frac{1}{4}$, $f'(3) = -\frac{1}{6}$. Tang: $y - \frac{1}{4} = -\frac{1}{6}(x-3)$

d.v.s: $y = -\frac{x}{6} + \frac{3}{4}$

2. $f(x) = \frac{x^2-4x+4}{x+1} = \left|\begin{matrix}\text{pol.}\\ \text{div.}\end{matrix}\right| = x - 5 + \frac{9}{x+1}$

D_f: $x \neq -1$. Asymp. $x = -1$ och $y = x - 5$.

Nollställen: $f(x) = \frac{(x-2)^2}{x+1}$. $x_{1,2} = 2$

$f'(x) = \frac{(x+1)(2x-4) - (x^2-4x+4)\cdot 1}{(x+1)^2} =$

$\frac{2x^2 \cancel{-4x} + 2x - 4 - x^2 \cancel{+4x} - 4}{(x+1)^2} =$

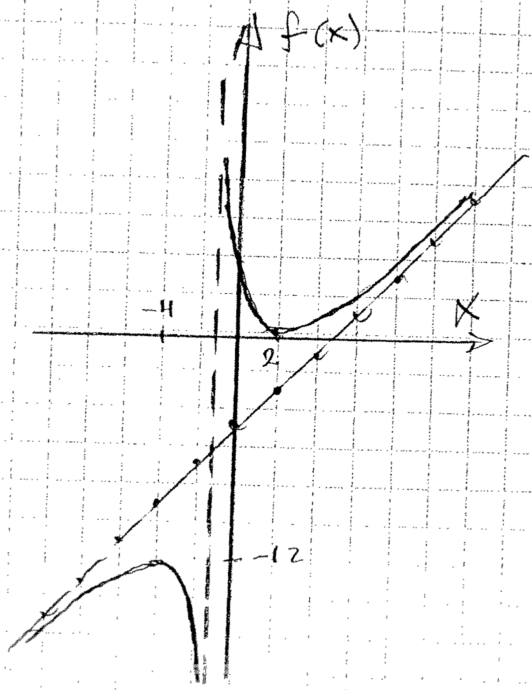
$= \frac{x^2+2x-8}{(x+1)^2}$, $f'(x) = 0 \Rightarrow$

$x^2 + 2x - 8 = 0$

$x = -1 \pm \sqrt{1+8} = -1 \pm 3$

$\begin{cases} x_1 = 2, & f(2) = 0 \\ x_2 = -4, & f(-4) = -12 \end{cases}$

x		-4		-1		2	
$f'(x)$	+	0	−		−	0	+
		lok maxp.				lok minp.	



Svar:

3. a) $\frac{x^3 e^{-2x}}{e^{-x}} = \frac{x^3}{e^x} \to 0$ då $x\to\infty$

b) $\frac{\sin 3x}{2x} = \underbrace{\frac{\sin 3x}{3x}}_{\to 1} \cdot \frac{3}{2} \to \frac{3}{2}$ då $x \to 0$

c) $\frac{2\ln 3x - \ln x}{x} = \frac{\ln 9x^2 - \ln x}{x} = \frac{\ln 9x}{x} \to 0$ då $x \to \infty$

4. a) $D\sqrt{x^2+2} = \dfrac{x}{\sqrt{x^2+2}}$ b) $D\,x^2\ln 3x = x^2\cdot\dfrac{3}{3x} + 2x\cdot\ln 3x = \underline{x+2x\ln 3x}$

c) $D\tan^2 2x = 2\tan 2x\cdot(1+\tan^2 2x)\cdot 2 = \underline{4\tan 2x\,(1+\tan^2 2x)}$

d) $D\,\dfrac{e^{2x}}{x} = \dfrac{x\cdot 2\cdot e^{2x} - e^{2x}\cdot 1}{x^2} = \underline{\dfrac{(2x-1)e^{2x}}{x^2}}$

5. $xy^2 + x^2 + 2y = 1$, $x=-3:\ -3y^2+9+2y-1=0$, $y^2-\frac{2}{3}y-\frac{8}{3}=0$

Impl. derivering:

$x\cdot 2yy' + y^2 + 2x + 2y' = 0$

$y = \frac{1}{3} \pm \sqrt{\frac{1}{9}+\frac{8\cdot 3}{3\cdot 3}} = \frac{1}{3}\pm\frac{5}{3}$ $y_1 = 2$, o.k! $y_2 = -\frac{4}{3}$ Nix!

$(-3,2):\ -12y' + 4 - 6 + 2y' = 0$

$-10y' = 2$, $y' = -\frac{1}{5}$

$k_T = -\frac{1}{5} \Rightarrow k_N = 5$.

Normals ekv. $y-2 = 5(x+3)$

dvs $\underline{y = 5x+17}$

6. $f(x) = xe^{-\sqrt{x}}$, $D_f:\ x \geq 0$. $\lim_{x\to\infty} xe^{-\sqrt{x}} = 0$, x-axeln asymp.

$f'(x) = \left[x\cdot\left(-\frac{1}{2\sqrt{x}}\right)+1\right]e^{-\sqrt{x}} = \left[1-\frac{\sqrt{x}}{2}\right]e^{-\sqrt{x}}$, $f'(x)=0 \Rightarrow x=4$, $f(4)=\frac{4}{e^2}$

$f''(x) = \left[\left(1-\frac{\sqrt{x}}{2}\right)\cdot\left(-\frac{1}{2\sqrt{x}}\right) - \frac{1}{4\sqrt{x}}\right]e^{-\sqrt{x}}$

$= \left[-\frac{1}{2\sqrt{x}} + \frac{1}{4} - \frac{1}{4\sqrt{x}}\right]e^{-\sqrt{x}} =$

$= \left[\frac{1}{4} - \frac{3}{4\sqrt{x}}\right]e^{-\sqrt{x}} = \frac{\sqrt{x}-3}{4\sqrt{x}}\cdot e^{-\sqrt{x}}$

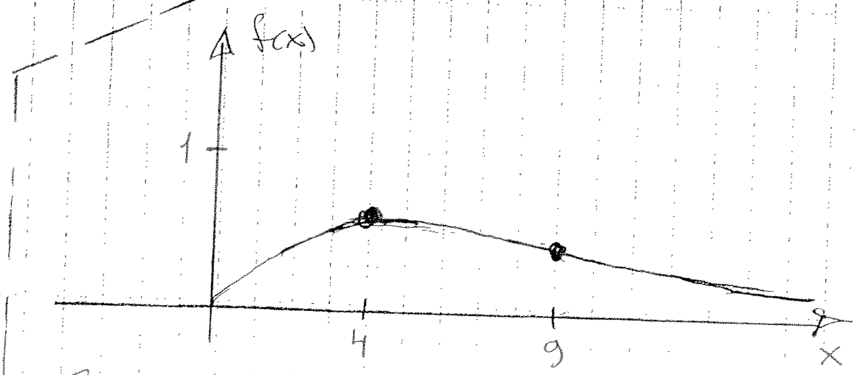
x		4	
$f'(x)$	+	0	−

lok.max.p.

$f''(x) = 0 \Rightarrow x = 9$

x		9	
$f''(x)$	−	0	+

teckenbyte!
Infl. punkt.



Svar: $D_f:\ x\geq 0$, Maxp $\left(4, \frac{4}{e^2}\right)$
Infl.pkt $\left(9, \frac{9}{e^3}\right)$

7. se boken.

8. — " —