Lösn. till mat. del D. Tekn. basår 2013-04-0

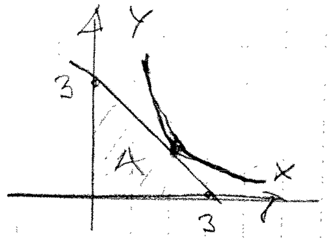
1. a) $\left[\frac{1}{x+h-1}-\frac{1}{x-1}\right]\cdot\frac{1}{h} = \frac{x-1-x-h+1}{(x+h-1)(x-1)h} = -\frac{1}{(x+h-1)(x-1)} \to -\frac{1}{(x-1)^2}$ då $h \to 0$

b) $f(2) = \frac{1}{2-1} = 1$. $f'(2) = -\frac{1}{(2-1)^2} = -1$.

Tang. ekv. $y-1 = -1(x-2)$

dvs. $y = -x+3$

$x=0 \Rightarrow y=3$. $y=0 \Rightarrow x=3$.



Svar: $A = 9/2$.

2. $f(x) = \frac{x^2+3x+5}{x-1}$, $D_f: x \neq 1$

Asymp. $x=1$

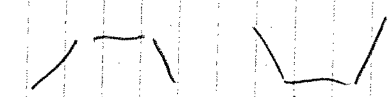
$f(x) = \frac{x^2+3x+5}{x-1} = \left\{\begin{matrix}\text{pol}\\ \text{div}\end{matrix}\right\} = \{\text{eller}\} = \frac{(x^2-x)+(4x-4)+9}{x-1} = x+4+\frac{9}{x-1}$

Asymp. $y = x+4$.

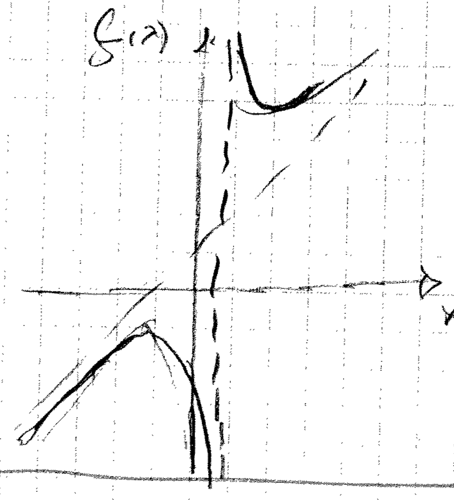
$f'(x) = 1 - \frac{9}{(x-1)^2}$

$f'(x) = 0 \Rightarrow (x-1)^2 = 9$, $x-1 = \pm 3$ $\begin{cases} x_1 = 4, & f(4) = 11 \\ x_2 = -2, & f(-2) = -1 \end{cases}$

x		-2		1		4	
$f'(x)$	$+$	0	$-$		$-$	0	$+$



lok max. lok min.



Svar:

3. a) $D\,2(1-3x)^{-1/2} = -(1-3x)^{-3/2}\cdot(-3) = \frac{3}{(1-3x)^{3/2}} = \frac{3}{(1-3x)\sqrt{1-3x}}$

b) $D\sin^2 5x = 2\sin 5x \cdot 5\cos 5x = 5\sin 10x$

c) $D\,\frac{x^2}{\ln 3x} = \frac{2x\cdot\ln 3x - x^2\cdot\frac{1}{x}}{(\ln 3x)^2} = \frac{2x\ln 3x - x}{(\ln 3x)^2}$

4. a) $\frac{3x^2}{\tan 2x^2} = \cos 2x^2 \cdot \frac{3x^2}{\sin 2x^2} = \underbrace{\cos 2x^2}_{\to 1} \cdot \underbrace{\frac{2x^2}{\sin 2x^2}}_{\to 1} \cdot \frac{3}{2} \to \frac{3}{2}$ då $x \to 0$

b) $\frac{x^3 e^{-x/2} \cdot e^{2x}}{e^{3x}} = \frac{x^2}{e^{3x/2}} \to 0$ (dominans) då $x \to \infty$

c) $\frac{8\ln x - 3\ln x^2}{x^2} = \frac{\ln x^8 - \ln x^6}{x^2} = \frac{\ln x^2}{x^2} = \frac{2\ln x}{x^2} = 0$ (dominans)

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5. $x^2y^2 + 6y - 5x + 6 = 0$. $x = 2$: $4y^2 + 6y - 10 + 6 = 0$

$y^2 + \frac{3y}{2} - 1 = 0$

$y = -\frac{3}{4} \pm \sqrt{\frac{9}{16} + \frac{16}{16}} = \frac{-3 \pm 5}{4}$

$y_1 = \frac{1}{2}$ (1:sta kvadr.) $y_2 = -2$ $P = (2, \frac{1}{2})$

$2xy^2 + 2x^2yy' + 6y' - 5 = 0$

$(2, \frac{1}{2})$: $1 + 4y' + 6y' - 5 = 0$

$10y' = 4$, $y' = \frac{2}{5} = k_T$

$k_N = -5/2$ Norm. ekv.:

$y - \frac{1}{2} = -\frac{5}{2}(x - 2)$ dvs $y = -\frac{5x}{2} + \frac{11}{2}$

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6. $f(x) = \sqrt{x}e^{-\sqrt{x}}$ D_f: $x \geq 0$. ($f(0) = 0$)

$\lim_{x \to \infty} \frac{\sqrt{x}}{e^{\sqrt{x}}} = 0$ (dominans) x-axeln asymp.

$f'(x) = \left[\sqrt{x}\left(-\frac{1}{2\sqrt{x}}\right) + \frac{1}{2\sqrt{x}}\right]e^{-\sqrt{x}} = \frac{1}{2}\left[\frac{1}{\sqrt{x}} - 1\right] \cdot e^{-\sqrt{x}}$

$f'(x) = 0 \Rightarrow x = 1$

x		1	
f'	+	0	−
		lok. max	

$f(1) = \frac{1}{e}$

$f''(x) = \frac{1}{2}\left[\left[\frac{1}{\sqrt{x}} - 1\right] \cdot \left(-\frac{1}{2\sqrt{x}}\right) - \frac{1}{2x\sqrt{x}}\right]e^{-\sqrt{x}}$

$= \frac{1}{2}\left[-\frac{1}{2x} + \frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}}\right] \cdot e^{-\sqrt{x}}$

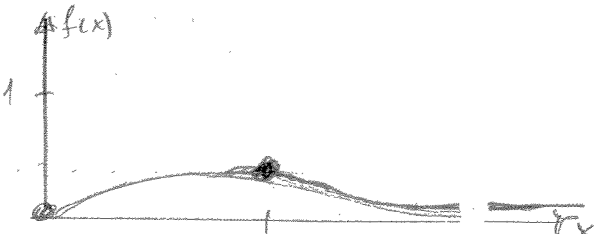
$= \frac{1}{4x\sqrt{x}}\left[-\sqrt{x} + x - 1\right]e^{-\sqrt{x}}$

$f'' = 0$

$x - \sqrt{x} - 1 = 0$ $t > 0$

$\sqrt{x} = t \Rightarrow t^2 - t - 1 = 0 \Rightarrow t = \frac{1 + \sqrt{5}}{2}$

$x = \left(\frac{1 + \sqrt{5}}{2}\right)^2 = \boxed{\frac{3 + \sqrt{5}}{2}}$ inflexionspunkten



x	0		$\frac{3+\sqrt{5}}{2}$		$+\infty$
$f''(x)$		−	0	+	