Matem analys 13/1-2010

1a $x^2+5=1-4x+4x^2 \Leftrightarrow 3x^2-4x-4=0$

$\Leftrightarrow x=\frac{2}{3}\pm\sqrt{\frac{4}{9}+\frac{4}{3}}=\frac{2}{3}\pm\frac{4}{3}$ $HL>0$

b $\frac{x(x+1)-(x^2-1)-6(x-1)}{(x-1)(x+1)}<0 \Leftrightarrow \frac{7-5x}{(x-1)(x+1)}<0$

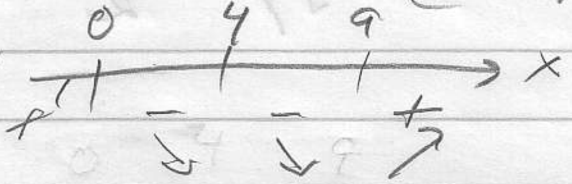
$+ \quad - \quad + \quad -$; $-1 \quad 1 \quad 7/5$; x

$-1<x<1$ el $\frac{7}{5}<x$

2a $\frac{x^2+5-(2x-1)^2}{(x-2)(\sqrt{x^2+5}+2x-1)}=\frac{-3(x-2)(x+2/3)}{(x-2)(\sqrt{x^2+5}+2x-1)}\to\frac{-8}{6}$

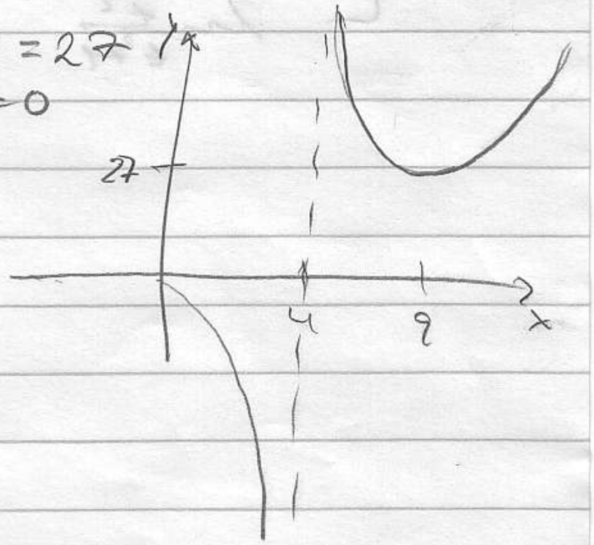
2b $\frac{x+2-5}{(x-3)(x+2)}\longrightarrow\frac{1}{5},\ x\to 3$

3 $f'=\frac{\frac{3}{2}x^{1/2}(x^{1/2}-2)-x^{3/2}\frac{1}{2}x^{-1/2}}{(\sqrt{x}-2)^2}=\frac{x-3\sqrt{x}}{(\sqrt{x}-2)^2}$



$f(9)=27$

$f(0)=0$



$f(x)/x=\frac{\sqrt{x}}{\sqrt{x}-2}\to 1=k,\ x\to+\infty$

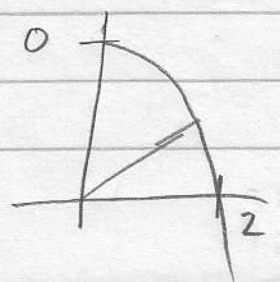
$f(x)-x=\frac{x\sqrt{x}-x\sqrt{x}+2x}{\sqrt{x}-2}\to\infty,\ x\to\infty$

4a $y^3+e^y=1 \Rightarrow y=0$

$3y^2y'+e^yy'=3x^2 \Rightarrow y'=3$

$6yy'^2+3y^2y''+e^yy'^2+e^yy''=6x \Rightarrow y''=6-9=-3$

4b



$d^2=x^2+(4-x^2)^2=16-7x^2+x^4=f(x)$

$f'(x)=-14x+4x^3=2x(2x^2-7)$

$0 \quad \sqrt{7/2}$; x

$(\sqrt{\frac{7}{2}},\frac{1}{2})$

5a $y' - \frac{1}{\sqrt{x}} y = 1 \quad \left(e^{-2\sqrt{x}} y\right)' = e^{-2\sqrt{x}}$

$e^{-2\sqrt{x}} y = \int e^{-2\sqrt{x}} dx + C = \left\{ \begin{array}{l} x = t^2 \\ dx = 2t\,dt \end{array} \right\} = \int 2t e^{-2t} dt$

$= -e^{-2t} \cdot t - \int -e^{-2t} \cdot 1\, dt = -te^{-2t} - \frac{1}{2} e^{-2t} + C$

$y = -\sqrt{x} - \frac{1}{2} + Ce^{2\sqrt{x}}$

5b $r^2 - 4 = 0 \quad y_h = C_1 e^{2x} + C_2 e^{-2x}$

$y_p = Ae^{3x} \quad y_p'' - 4y_p = 8Ae^{3x} = 2e^{3x} \quad A = 1/4$

6a $x^2 + 2 = t \quad \int 1 \ln t\, dt = t\ln t - \int t \cdot \frac{1}{t}\, dt$

$= t\ln t - t + C = (x^2+2)\ln(x^2+2) - x^2 - 2 + C$

6b $\frac{A}{t} + \frac{Bt + C}{t^2+1} = \frac{(A+B)t^2 + Ct + A}{t(t^2+1)} \quad \int \left(\frac{2}{t} + \frac{-2t+1}{t^2+1}\right) dt$

$= \left[\underbrace{2\ln|t| - \ln(t^2+1)}_{\ln \frac{t^2}{t^2+1}} + \arctan t\right]_1^\infty = \frac{\pi}{2} + \ln 2 - \frac{\pi}{4}$