SVAR Matem Analys 10/3 - 2009
(OBS! Ej fullst. (ŌBn))

1a) $1-6x+9x^2=4x \iff 9\left(x^2-\frac{10}{9}x+\frac{1}{9}\right)=0$

$\iff 9\left(\left(x-\frac{5}{9}\right)^2+\frac{1}{9}-\frac{25}{9^2}\right)=0 \iff x=\frac{5}{9}\pm\frac{4}{9}$

$\cancel{x=1 \Rightarrow VL<0}$ $\quad \underline{\underline{x=\frac{1}{9}}} \Rightarrow VL>0$

1b) $\frac{(x-4)(3x+1)-(x-8)(x-1)}{(x-1)(3x+1)}\geq 0 \iff \frac{2x^2-2x-12}{(x-1)(3x+1)}\geq 0$

$\iff \frac{2\left(\left(x-\frac{1}{2}\right)^2-\frac{25}{4}\right)}{(x-1)(3x+1)}\geq 0$

$-2 \quad -1/3 \quad 1 \quad 3 \quad x$

$+ \quad - \quad + \quad - \quad +$

SVAR $x\leq -2$ eller $-\frac{1}{3}<x<1$ eller $x\geq 3$

2a) $\frac{x^2-x+2-2x}{x(x-2)(\sqrt{x^2-x+2}+\sqrt{2x})}=\frac{\left(x-\frac{3}{2}\right)^2-\frac{1}{4}}{x(x-2)(\sqrt{\ }+\sqrt{\ })}=\frac{(\cancel{x-2})(x-1)}{x(\cancel{x-2})(\sqrt{\ }+\sqrt{\ })}$

$\to \frac{1}{2\cdot 4}, x\to 2$

2b) $\frac{(-2x+5)^2-(4x^2+2x+1)}{-2x+5+\sqrt{4x^2+2x+1}}=\frac{\overbrace{-22x+24}^{x\left(-22+\frac{24}{x}\right)}}{x\left(-2+\frac{5}{x}-\sqrt{4+\frac{2}{x}+\frac{1}{x^2}}\right)}\to\frac{-22}{-4}=\frac{11}{2},$ $x\to-\infty$

2c) $e^t=1+t+\frac{t^2}{2}+O(t^3)$ enl. Taylor

$t=-\frac{x}{2} \Rightarrow e^{-\frac{x}{2}}=1-\frac{x}{2}+\frac{x^2}{8}+O(x^3)$

$\Rightarrow \frac{x^2}{e^{-\frac{x}{2}}-1+\frac{x}{2}}=\frac{x^2}{\frac{x^2}{8}+O(x^3)}=\frac{1}{\frac{1}{8}+O(x)}\to 8, x\to 0$

3) $f'(x) = \dfrac{(\ln x)^2 - x \cdot 2\ln x \cdot \frac{1}{x}}{(\ln x)^4} = \dfrac{\ln x - 2}{(\ln x)^3}$ ②

$D_f: x > 0,\ x \neq 1$

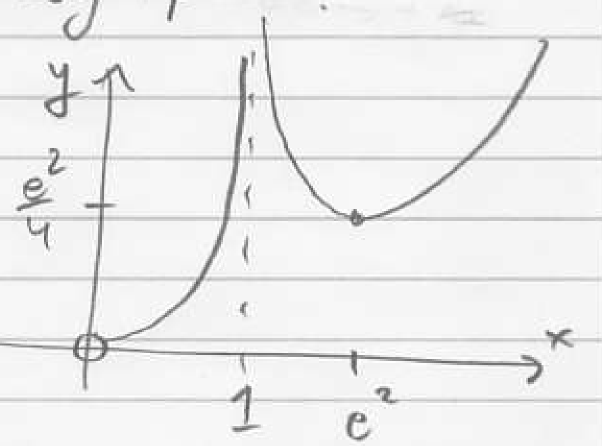
x	0		1		e^2	
f'		$+$		$-$		$+$

$\dfrac{x}{(\ln x)^2} \to \begin{cases} +\infty, & x \to 1 \\ 0, & x \to 0^+ \end{cases}$

$\dfrac{f(x)}{x} = \dfrac{1}{(\ln x)^2} \to 0,\ x \to \infty \qquad f(x) - 0 \cdot x = \dfrac{x}{(\ln x)^2} \to \dfrac{\infty}{\infty},$ $x \to \infty$ dominerar

Alltså finns ej sned asymptot.

$f(e^2) = \dfrac{e^2}{2^2} = \dfrac{e^2}{4}$



4a) $\underline{f(1) = 0}$

$e^{-2f} 2f'(f^2+1) + e^{-2f} \cdot 2ff' = 1$

$x=1 \Rightarrow \underline{f'(1) = 1/2}$

$f'(f^2+f+1) = e^{2f}/2$

$\Rightarrow f''(f^2+f+1) + f'(2ff'+f') = e^{2f} \cdot f'$

$x=1 \Rightarrow \quad f'' + \frac{1}{4} = \frac{1}{2} \quad \Rightarrow \quad \underline{f''(1) = \frac{1}{4}}$

4b) $D\, e^{-2x}(x^2+1) = e^{-2x}\left(-2(x^2+1) + 2x\right)$

$= -2\underbrace{e^{-2x}}_{>0}\underbrace{\left(\left(x-\frac{1}{2}\right)^2 + \frac{3}{4}\right)}_{>0} < 0$

$\Rightarrow$ str. avt. $\Rightarrow$ entydigt lösbar

5a) $y' - \frac{1}{\sqrt{x}} y = \sqrt{x}$ int. faktor $e^{-2\sqrt{x}}$

$\Leftrightarrow \quad y' e^{-2\sqrt{x}} \underbrace{- \frac{1}{\sqrt{x}} e^{-2\sqrt{x}}}_{D e^{-2\sqrt{x}}} y = \sqrt{x} e^{-2\sqrt{x}}$

$\Leftrightarrow$

$D\left(y \cdot e^{-2\sqrt{x}}\right) = \sqrt{x}\, e^{-2\sqrt{x}}$

$\Leftrightarrow \quad y e^{-2\sqrt{x}} = \int \sqrt{x} e^{-2\sqrt{x}} dx = \left\{ \begin{array}{l} x = t^2 \\ dx = 2t\,dt \end{array} \right\} = \int 2t^2 e^{-2t} dt + C$

$= -e^{-2t} t^2 + \int e^{-2t} 2t\, dt = -e^{-2t} t^2 - e^{-2t} \cdot t + \underbrace{\int e^{-2t} dt}_{-\frac{1}{2} e^{-2t}}$

$y = -x - \sqrt{x} - \frac{1}{2} + C e^{2\sqrt{x}}$

5b) y_h: $r^2 + 4 = 0 \quad r = \pm 2i \Rightarrow y_h = A \cos 2x + B \sin 2x$

$y_{p_1} = (ax+b) e^{-x} \Rightarrow P(D) y_{p_1} = e^{-x} ((D-1)^2 + 4)(ax+b)$

$= e^{-x}(D^2 - 2D + 5)(ax+b) = e^{-x}(-2a + 5ax + 5b) = x e^{-x} \quad \left\{ \begin{array}{l} 5a = 1 \\ -2a + 5b = 0 \end{array} \right.$

$a = 1/5 \quad b = 2a/5 = 2/25 \quad y_{p_1} = \frac{1}{25}(5x+2) e^{-x}$

$y_{p_2} = cx \cos 2x + dx \sin 2x$

$y_{p_2}' = c \cos 2x - 2cx \sin 2x + d \sin 2x + 2dx \cos 2x$

$y_{p_2}'' = -2c \sin 2x - 2c \sin 2x - 4cx \cos 2x + 2d \cos 2x + 2d \cos 2x - 4dx \sin 2x$

$y_{p_2}'' + 4 y_{p_2} = \cos 2x(\cancel{4cx} - \cancel{4cx} + \underbrace{2d + 2d}_{4d}) + \sin 2x(\cancel{4dx} \underbrace{- 2c - 2c}_{-4c} - \cancel{4dx})$

$= 2 \cos 2x \qquad 4d = 2, \; -4c = 0$

$c = 0, \quad d = \frac{1}{2} \Rightarrow y_{p_2} = \frac{1}{2} x \sin 2x$

$y = y_h + y_{p_1} + y_{p_2} = A \cos 2x + \left(\frac{x}{2} + B\right) \sin 2x + \frac{1}{25}(5x+2) e^{-x}$

6a) $\int \frac{\cos x}{3+\sin^2 x}\,dx = \left\{ \begin{array}{l} \sin x = t \\ \cos x\,dx = dt \end{array} \right\}$

$$= \int \frac{dt}{3+t^2} = \frac{1}{\sqrt{3}} \arctan\left(\frac{t}{\sqrt{3}}\right)$$

$$= \frac{1}{\sqrt{3}} \arctan\left(\frac{\sin x}{\sqrt{3}}\right) + C$$

6b) $\frac{1}{2}\int_1^\infty \frac{1}{\left(\left(x+\frac{3}{4}\right)^2 - \frac{25}{4^2}\right)}\,dx = \frac{1}{2}\int_1^\infty \left(\frac{a}{x-\frac{1}{2}} + \frac{b}{x+2}\right)dx$

$1 = a(x+2) + b\left(x-\frac{1}{2}\right) \qquad a+b=0,\ 2a-\frac{b}{2}=1$

$a = \frac{2}{5} \quad b = -\frac{2}{5}$

$$= \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{5}\left[\ln\left|\frac{x-\frac{1}{2}}{x+2}\right|\right]_1^\infty = \frac{1}{5}\left(\ln 1 - \ln\frac{1}{6}\right) = \underline{\underline{\frac{1}{5}\ln 6}}$$