Uppgift 1(a).

```
function udot=odefun(t,u)
if t<0.003
    q=10000;
else
    q=0;
end
udot=q-(u^4-1);
```

(b).

```
T=0.01; u0=1;
tspan=linspace(0,T,100);
[t,U]=ode45(@odefun,tspan,u0);
plot(t,U)
```

Uppgift 2(a). Inför $x_i = ih, i = 0, 1, \cdots, n+1$, med $h = \frac{L}{n+1}$.

$$-u''(x_i) + 5u(x_i) = \lambda u(x_i),\ i = 1, 2, \cdots, n$$

$$-\frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} + 5u_i = \lambda u_i,\ i = 1, 2, \cdots, n$$

$$-u_{i-1} + (2 + 5h^2)u_i - u_{i+1} = h^2 \lambda u_i,\ i = 1, 2, \cdots, n$$

Randvillkor: $u'(0) + u(0) = 0$ ger $\frac{u_1 - u_0}{h} + u_0 = 0$, dvs. $u_0 = \frac{1}{1-h} u_1$, $u(L) = 0$ ger $u_{n+1} = 0$.
Matrisformulering med $c_1 = 2 + 5h^2 - \frac{1}{1-h}$ och $c_i = 2 + 5h^2$, $i = 2, \cdots, n$, lyder

$$\begin{bmatrix} c_1 & -1 & & & \\ -1 & c_2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & c_{n-1} & -1 \\ & & & -1 & c_n \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{n-1} \\ u_n \end{bmatrix} = h^2 \lambda \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{n-1} \\ u_n \end{bmatrix}$$

(b). Se laboration 1.

(c).

```
n=50; L=1; k=3;
h=L/(n+1); x=h*[0:n+1]'; e=ones(n,1);
A=spdiags([-e (2+5*h^2)*e -e],[-1 0 1],n,n); A(1,1)=A(1,1)-1/(1-h);
[V,D]=eigs(A,k,'SM');
V=[V(1,:)/(1-h);V;zeros(1,k)]; lambda=diag(D)/h^2;
for j=1:k
    subplot(3,1,j), plot(x,V(:,j))
    title(['\lambda = ',num2str(lambda(j))])
end
```

Uppgift 3. Se laboration 4 eller Lay kapitel 5.

Uppgift 4(a).

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} x_1^3 + x_2 - 1 \\ x_1 + x_2 \exp(x_1 x_2) + 1 \end{bmatrix}, \quad \boldsymbol{Df}(\boldsymbol{x}) = \begin{bmatrix} 3x_1^2 & 1 \\ 1 + x_2^2 \exp(x_1 x_2) & (1 + x_1 x_2) \exp(x_1 x_2) \end{bmatrix}$$

(b). Se laboration 5.

(c). Newtons metod: $\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \boldsymbol{d}_k$, där $\boldsymbol{Df}(\boldsymbol{x}_k)\,\boldsymbol{d}_k = -\boldsymbol{f}(\boldsymbol{x}_k)$

$$\boldsymbol{x}_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} : \quad \boldsymbol{Df}(\boldsymbol{x}_0)\,\boldsymbol{d}_0 = -\boldsymbol{f}(\boldsymbol{x}_0) \Leftrightarrow \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} d_{01} \\ d_{02} \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix} \Leftrightarrow \boldsymbol{d}_0 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\boldsymbol{x}_1 = \boldsymbol{x}_0 + \boldsymbol{d}_0 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

(d).

```
f1=@(x1,x2)x1.^3+x2-1;
f2=@(x1,x2)x1+x2.*exp(x1.*x2)+1;
f=@(x)[f1(x(1),x(2));f2(x(1),x(2))];
Df=@(x)[3*x(1)^2 1; 1+x(2)^2*exp(x(1)*x(2)) (1+x(1)*x(2))*exp(x(1)*x(2))];
x=[0;1];
kmax=10; tol=0.5e-8;
for k=1:kmax
    d=-Df(x)\f(x);
    x=x+d;
    disp([x' norm(d)])
    if norm(d)<tol, break, end
end
```

Uppgift 5. $f(\boldsymbol{x}) = x_1^2 - 3x_1 + x_1 x_2 + x_2^2$, $\quad \boldsymbol{\nabla} f(\boldsymbol{x}) = \begin{bmatrix} 2x_1 - 3 + x_2 \\ x_1 + 2x_2 \end{bmatrix}$

(a). $\boldsymbol{x}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\quad \boldsymbol{\nabla} f(\boldsymbol{x}_0) = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$

$$\boldsymbol{x}(s) = \boldsymbol{x}_0 - s\boldsymbol{\nabla} f(\boldsymbol{x}_0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - s \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 - 3s \end{bmatrix}$$

$$g(s) = f(\boldsymbol{x}(s)) = f(1, 1 - 3s) = 1 - 3 + (1 - 3s) + (1 - 3s)^2 = -9s + 9s^2$$

$$g'(s) = -9 + 18s = 0 \Leftrightarrow \hat{s} = \tfrac{1}{2}$$

$$\boldsymbol{x}_1 = \boldsymbol{x}_0 - \hat{s}\,\boldsymbol{\nabla} f(\boldsymbol{x}_0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \hat{s} \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -\frac{1}{2} \end{bmatrix}$$

(b).

```
f=@(x1,x2)x1.^2-3*x1+x1.*x2+x2.^2;
funf=@(x)f(x(1),x(2));
dfdx1=@(x1,x2)2*x1-3+x2; dfdx2=@(x1,x2)x1+2*x2;
gradf=@(x)[dfdx1(x(1),x(2));dfdx2(x(1),x(2))];
kmax=100; smax=10; tol=1e-4;
x=[1;1];
for k=1:kmax;
    grad=gradf(x);
    g=@(s)funf(x-s*grad);
```

```
    sk=fminbnd(g,0,smax);
    x=x-sk*grad;
    disp([x' funf(x)])
    if norm(sk*grad)<tol, break, end
end
```

Uppgift 6(a). Inför nät med steglängd h i rumsled och ersätter u''_{xx} med D_+D_- Låter $u_i(t)$ beteckna approximationen av $u(x_i,t)$.

Får följande begynnelsevärdesproblem för ODE-system.

$$\begin{cases} u_i''(t) = \frac{u_{i+1}(t)-2u_i(t)+u_{i-1}(t)}{h^2}, i=1,\cdots,n, 0<t<T \\ u_0(t)=0, u_{n+1}(t)=\sin(3t) \\ u_i(0)=\sin(2\pi x_i), i=1,\cdots,n \\ u_i'(0)=0, i=1,\cdots,n \end{cases}$$

På vektorform:

$$\begin{cases} \boldsymbol{U}''(t) = \frac{1}{h^2}(\boldsymbol{b}(t)+\boldsymbol{A}\boldsymbol{U}(t)), 0<t<T \\ \boldsymbol{U}(0)=\boldsymbol{U}_0,\ \boldsymbol{U}'(0)=\boldsymbol{V}_0 \end{cases}$$

där $\boldsymbol{A}$ är diskreta motsvarigheten till u'' och

$$\boldsymbol{U}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_{n-1}(t) \\ u_n(t) \end{bmatrix} \quad \boldsymbol{b}(t) = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \sin(3t) \end{bmatrix} \quad \boldsymbol{U}_0 = \begin{bmatrix} \sin(2\pi x_1) \\ \sin(2\pi x_2) \\ \vdots \\ \sin(2\pi x_{n-1}) \\ \sin(2\pi x_n) \end{bmatrix} \quad \boldsymbol{V}_0 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Slutligen får vi formulera om vår ODE som ett första ordningens system.

$$\begin{cases} \boldsymbol{W}'(t) = \boldsymbol{f}(t,\boldsymbol{W}), 0<t<T \\ \boldsymbol{W}(0)=\boldsymbol{W}_0 \end{cases}$$

där

$$\boldsymbol{W} = \begin{bmatrix} \boldsymbol{U} \\ \boldsymbol{V} \end{bmatrix}, \quad \boldsymbol{f}(t,\boldsymbol{W}) = \begin{bmatrix} \boldsymbol{V}(t) \\ \frac{1}{h^2}(\boldsymbol{b}(t)+\boldsymbol{A}\boldsymbol{U}(t)) \end{bmatrix}, \quad \boldsymbol{W}_0 = \begin{bmatrix} \boldsymbol{U}_0 \\ \boldsymbol{V}_0 \end{bmatrix}$$

(b).

```
T=5; L=1; u0=@(x)sin(2*pi*x);
n=30; h=L/(n+1); xi=h*[1:n]'; tspan=linspace(0,T,n+1);
U0=u0(xi); V0=zeros(size(xi)); W0=[U0; V0];
A=spdiags(ones(n,1)*[-1 2 -1],[-1 0 1],n,n);
b=@(t)[0;zeros(n-2,1);sin(3*t)];
odefun=@(t,w)[w(n+1:2*n);1/h^2*(b(t)-A*w(1:n))];
[t,W]=ode45(odefun,tspan,W0);
x=[0;xi;L]; U=[zeros(size(t)),W(:,1:n),sin(3*t)];  % Kantar med randvärden
surf(x,t,U)
xlabel('x'), ylabel('t')
```