Facit, TMA132 Fourieranalys F2/Kf2, 5 poäng

980820

1.

$$f(x) = \frac{a}{2\pi} + \frac{4}{\pi a} \sum_{n=1}^{\infty} \frac{\sin^2(na/2)}{n^2} \cos(nx).$$

och

$$\sum_{n=1}^{\infty} \frac{\sin^2(na/2)}{n^2} = \frac{a}{4}(\pi - \frac{a}{2}), \qquad \sum_{n=1}^{\infty} \frac{\sin^4(na/2)}{n^4} = \frac{a^2}{16}\Big(\frac{2a\pi}{3} - \frac{a^2}{2}\Big).$$

2.

$$u(x,t) = 2x - \frac{2}{\pi^2} \sum_{n=0}^{\infty} \frac{(n+1/2)\pi + (-1)^n}{(n+1/2)^2} e^{-k(n+1/2)^2\pi^2 t} \sin(n+1/2)\pi x.$$

3.

$$\int_{-\infty}^{\infty} \hat{f}(\xi)\ \cos(\xi)\, d\xi = \frac{\pi}{2} \cosh 1.$$

4.

$$u(x,t) = \sum_{n=0}^{\infty} 2^{-n-1} e^{-nt} L_n(x).$$

5.

$$u(r,\theta) = -\frac{8}{\pi} \sum_{n=1,3,5,\ldots} \frac{1}{n(n^2-4)} \frac{J_n(kr)}{J_n(k)} \sin(n\theta);$$

Villkor: $J_n(k) \neq 0$ för udda n.

990116

1.

$$\int_{\infty}^{\infty} F(\xi) \cos(\xi)\, d\xi = \frac{\pi}{4}.$$

2.

$$\int_{\infty}^{\infty} h(t) \frac{\sin t}{t}\, dt = \frac{\pi}{4} - \frac{1}{2}.$$

3.

$$u(x,y) = y + \sum_{n=1}^{\infty} \frac{2}{n\pi \cosh n\pi} \Big[\frac{1-(-1)^n}{n\pi} \sinh n\pi x + (-1)^n \cosh n\pi(1-x) \Big] \sin(n\pi y).$$

4.

$$P(x) = \frac{2}{35}(3 + 24x - 10x^2).$$

5.

$$u(r,t) = \sin t + \sum_{n=1}^{\infty} \frac{2}{\alpha_n J_1(\alpha_n)} \frac{1}{(\frac{\alpha_n}{b})^4 + 1} \Big((\frac{\alpha_n}{b})^2 e^{-(\frac{\alpha_n}{b})^2 t} - \sin t - (\frac{\alpha_n}{b})^2 \cos t \Big) J_0(\frac{\alpha_n r}{b}).$$

990309

1.

$$y(t) = \frac{4\pi}{27} + \sum_{n \neq 0} \frac{2(2i-n)}{n(3+in)^3} e^{int}.$$

2.

$$u(x,t) = 3 - x^2 + \frac{12}{\pi^3} \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(n+1/2)^3} e^{-\frac{1}{2}(n+1/2)^2 \pi^2 t} \cos(n+1/2)\pi x.$$

3.

$$u(x,t) = \frac{e^{1/2}(2t+x)}{(2t+x)^{3/2}} e^{-\frac{(x-1)^2}{2(2t+1)}}.$$

4.

$$u(r,\theta) = 1 + \frac{3}{5}(R^2 - x^2 - y^2) + \frac{2}{5} z^3.$$

990819

1.

$$\cos(\omega_0 t) \curvearrowright \frac{(4-\omega_0)^2 \cos \omega_0 t + 4\omega_0 \sin \omega_0 t}{(4+\omega_0^2)^2}.$$

2.

$$u(x,t) = \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{n(-1)^{n-1}}{(n^2 - 1/4)^2} \cdot e^{-(1+n^2\pi^2)t} \sin n\pi x.$$

3.

$$e^{-x} = \sum_{n=0}^{\infty} \frac{2}{(n+1/2)\pi} e^{-x} \sin(n+1/2)\pi x.$$

4.

$$u(x,y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\sinh \xi y}{\xi \cosh \xi} \cdot \hat{f}(\xi)\, e^{i\xi x}\, d\xi.$$

010830

1.

$$a) \quad \int_{-\infty}^{\infty} |f(x)|^2\, dx = 4\pi/3, \quad \int_{-\infty}^{\infty} f(x) \cos(x)\, dx = \pi/2.$$

2.

$$u(x,y) = \frac{1}{6}(y^3 - y) + \sum_{n=1}^{\infty} \frac{14(-1)^n}{n^3\pi^3} \sin(n\pi y).$$

3.

$$|\theta| \sim \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2n-1)\theta}{(2n-1)^2}, \quad y(\theta) = \frac{\pi}{4} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2n-1)\theta}{(2n-1)^2((2-(2n-1)^2)}.$$

4.

$$\sum_{k=-\infty}^{\infty} |\hat{u}_k(\xi)|^2 = 1.$$

5.

$$u(r,\theta) = \frac{4C}{\pi} \sum_{n=1}^{\infty} \frac{1}{2k-1} \frac{\sinh \frac{(2k-1)\pi\theta}{\ln a}}{\sinh \frac{(2k-1)\pi^2}{2\ln a}} \cdot \sin \frac{(2k-1)\pi \ln r}{\ln a}.$$

6.

$$\Phi(x,y) = \frac{\Phi_0}{\pi} arccot \frac{2y}{1 - x^2 - y^2}.$$