

# **Exam for MVE041 och MMGL32 Flervariabelmatematik**

**The 28 August 2015, kl. 830-1230**

Help materials: Attached formula sheet. No calculators.

Telephone: \*\* \*\*, \*\*-\*\*

Total points are 50. Passing this course requires a) 25 points of 32 points on the *Passing Part*, and b) a pass on all six Matlab labs. Your bonus points from this course apply to the passing part of the exam. The maximum score on the passing part is 32. A grade of 4 or 5 is obtained with scores of 33, and 42 respectively. Bonus points do not apply to the mastery part of the exam.

Solutions will be posted on the course website on the first weekday following the exam. The exam is graded anonymously. Results are available on Ladok starting three weeks after the exam day. The first day on which you may contest your grade will be posted on the course website, and after that you may file a contest with the MV exp any week day 9-13.

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## **Passing Part**

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## **Mastery Part**

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# Equations and Formulas MVE041, 14/15

## Geometri

Circle radius  $a$ : Area =  $\pi a^2$ .

Sphere of radius  $a$ : Vol. =  $\frac{4}{3}\pi a^3$ , Area =  $4\pi a^2$

Cylinder radius  $a$ , height  $h$ : Vol. =  $\pi a^2 h$ , Area =  $2\pi a h$ .

Circular cone rad.  $a$ , height  $h$ . Vol. =  $1/3\pi a^2 h$

## Trigonometri

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cos(2x) = 1 - 2 \sin^2(x)$$

$$\cos(x+y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

$$\sin(x+y) = \sin(x) \cos(y) + \cos(x) \sin(y)$$

$$\cos(x) \cos(y) = \frac{1}{2}(\cos(x-y) + \cos(x+y))$$

$$\sin(x) \sin(y) = \frac{1}{2}(\cos(x-y) - \cos(x+y))$$

$$\sin(x) \cos(y) = \frac{1}{2}(\sin(x-y) + \sin(x+y))$$

$$\cos^2(x) + \sin^2(x) = 1$$

## Derivater

$$\frac{d}{dx} x^a = ax^{a-1},$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \cos(x) = -\sin(x)$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}, x > 0$$

$$\frac{d}{dx} \sin(x) = \cos(x)$$

$$\mathbf{grad} f(x, y, z) = \bar{\nabla} f(x, y, z), \quad \mathbf{div} \bar{\mathbf{F}}(x, y, z) = \bar{\nabla} \cdot \bar{\mathbf{F}}(x, y, z), \quad \mathbf{curl} \bar{\mathbf{F}}(x, y, z) = \bar{\nabla} \times \bar{\mathbf{F}}(x, y, z)$$

## Integrals

$$\int x^a dx = \frac{x^{a+1}}{a+1} + C$$

$$\int \frac{1}{x} dx = \ln|x| + C,$$

$$\int \sin(x) dx = -\cos(x) + C,$$

$$\int \cos(x) dx = \sin(x) + C$$

$$\int e^x dx = e^x + C,$$

$$\int \ln(x) dx = x \ln(x) - x + C,$$

$$\int \sin^2(x) dx = \frac{x}{2} - \frac{1}{4} \sin(2x) + C,$$

$$\int \cos^2(x) dx = \frac{x}{2} + \frac{1}{4} \sin(2x) + C,$$

$$\int \sin^3(x) dx = -\frac{1}{3} \sin^2(x) \cos(x) - \frac{2}{3} \cos(x) + C,$$

$$\int \cos^3(x) dx = \frac{1}{3} \cos^2(x) \sin(x) + \frac{2}{3} \sin(x) + C,$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = -\sin^{-1}\left(\frac{x}{a}\right) + C, \quad (a > 0, |x| < a)$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C, \quad (a > 0)$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{x+a}{x-a} \right| + C, \quad (a > 0)$$

$$\int \frac{dx}{x\sqrt{a^2 - x^2}} = \frac{1}{a} \sec^{-1}\left|\frac{x}{a}\right| + C, \quad (a > 0, |x| > a)$$

## Maclaurinutvecklingar

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$\ln(1+x) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k}$$

$$\sin x = \sum_{k=1}^{\infty} (-1)^{k-1} \frac{x^{2k-1}}{(2k-1)!}$$

$$\cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$$

$$(1+x)^\alpha = \sum_{k=0}^{\infty} \frac{\alpha!}{k!(\alpha-k)!} x^k$$

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$