MVE041 Flervariabelanalys 2015

This document contains the learning goals for this course. The goals are organized by subject, with reference to the course textbook “Calculus: A Complete Course” 8th ed. by Adams and Essex. Specific expected knowledge at the passing and mastery level is specified for each topic.

Contents

1 Functions of n-variables

1.1 Euclidean Geometry in $\mathbb{R}^n$, §10.1 and §10.5

In which studied concepts of Euclidean geometry in $\mathbb{R}^n$.

1.1.1 Passing

- Compute the distance between points in $\mathbb{R}^n$, and points and lines or planes in $\mathbb{R}^n$.
- Describe, write the equations for, and sketch representations of subsets of $\mathbb{R}^3$ including planes, cylinders, spheres, cones, ellipsoids, paraboloids, and hyperboloids, as well related solids.
- Understand the meaning and definition of open and closed balls in $\mathbb{R}^n$.

- Understand the concepts of open set, closed set, complement, as well as interior, exterior and boundary points.

1.2 Representations of functions of n-variables, §12.1 - §12.3

In which we studied representations of functions such as graphs and level curves.

1.2.1 Passing

- Understand the concept of a graph of a function of n variables. Given a function, be able to write as a set, and sketch the graph of the function in an appropriate domain.
- Understand the concepts of domain and range of a function. Given a function, be able to specify its domain and range.
- Understand concept of level surfaces. Given a function be able to describe and sketch specified or representative level surfaces or curves. Understand the relation between the level surfaces and a graph of a function.

2 Derivatives of Functions of n-Variables

2.1 Limits and Partial Derivatives, §12.1 - §12.4

2.1.1 Passing

In which we studied limits and partial derivatives, the tangent plane and normal line at points.

- Be able to give an intuitive description of the limit of a function of two variables. Evaluate the limit of such a function using the usual laws of limits.
- Know the definition of a continuous function.
- Given a function, be able to compute its partial derivatives, and evaluate them at a point. Be familiar with the different notation for partial derivatives.
- Understand and sketch a tangent plane and normal vector to the graph of a function at a point. Given a function, be able to compute the equation of the tangent plane and normal vector at a specified point, and write the parametrized equation for the normal line through that point.
- Compute n-th order partial derivatives of a function.

2.1.2 Mastery

- Be able to state and motivate the definition of a limit of a function of two variables.
- Be able to evaluate a specified limit, and if appropriate show that it exists.
- Determine whether a function is continuous.
- Know and motivate the definition of a partial derivative for a scalar-valued function of two variables.
- Show that a function is harmonic.

2.2 The Chain Rule, §12.5

2.2.1 Passing

- Use appropriate versions of the chain rule to compute the partial derivatives of functions of the form
$$z = f(x, y), \quad \text{where } x = u(t), \text{ and } y = v(t),$$
and
$$z = f(x, y), \quad \text{where } x = u(t, s), \text{ and } y = v(t, s).$$

2.3 Linear Approximations, §12.6

2.3.1 Passing

- Be able to use the tangent plane at a point to approximate the value of a function at a nearby point.
- Use the differential of a function at a point to estimate the function value at a nearby point, and to approximate percentage errors.
- Compute the linearization of a vector-valued function of a vector variable at a point (i.e. using the Jacobian derivative).

2.3.2 Mastery

- Give the definition of a differentiable function of two variables (Def. 5 of *Adams and Essex*).
- State the mean value theorem for a real-valued function of two real variables (Theorem 3 in 12.6 of *Adams and Essex*).
- Understand the relationship between continuity, existence of partial derivatives, and differentiability for a function. What property of the first partial derivatives of a function guarantees that the function is differentiable at a point (cf. Theorem 4 in 12.6 of *Adams and Essex*).

2.4 Gradients and Directional Derivatives, §12.7

2.4.1 Passing

- Understand and be able to give the definition of the gradient of a function of n-real variables. Given such a function be able to compute its gradient.
- Know and understand the geometric properties of the gradient vector.
- Be able to compute the directional derivative of a function of two real variables in the direction of a specified vector and evaluate this at a point.
- Compute the equations of tangent line and normal line to a level curve as well as tangent plane and normal line to a level surface at a point.

2.4.2 Mastery

- For a real-valued function of two real variables be able to prove that the gradient vector of a function at a point is orthogonal to the level curve of the function at that point (cf. Theorem 6 of 12.7 of *Adams and Essex*).
- Be able to state the definition of the directional derivative. Prove that the directional derivative can be computed using the gradient (cf. Theorem 7 of 12.7 of *Adams and Essex*).

2.5 Taylor Series, §12.9

2.5.1 Passing

- Be able to write down the Taylor series for a general real-valued function of n real variables.
- Explicitly compute the Taylor series for a specified real-valued function of two real variables using the Taylor formula as well as using known expressions for functions of one variable.

2.6 Implicit Functions, §12.8

2.6.1 Mastery

- Understand the relationship between the Jacobian determinant and transformations of $\mathbb{R}^n$. Be able to state the implicit function theorem for one equation in three variables.
- Be able to compute specified partial derivatives of functions implicitly defined by equations of two or three variables. Also specify the domain on which these expressions are valid.

3 Integrals of Functions of n-Variables

3.1 Double Integrals over Regular Domains, §14.1 - §14.2, §14.4

3.1.1 Passing

- Use a double integral and cartesian coordinates to find the area of a bounded region in the plane.
- Compute the double integral in cartesian coordinates of a function over a bounded region in the plane.
- Compute the double integral in polar coordinates of a function defined on a bounded region in the plane.
- Know the transformations between cartesian and polar coordinates as well as the area element in both coordinate systems. Be able to compute the area element in polar coordinates using the Jacobian determinant.
- Understand what it means for a map $\mathbb{R}^2 \to \mathbb{R}^2$ to be one-to-one.
- Can compute the area element in a new system of coordinates, for given variable transformations.

3.1.2 Mastery

- Define and use a suitable variable substitution to compute a given double integral.

3.2 Improper Double Integrals, §14.3

3.2.1 Passing

- Determine whether a double integral is improper and if so, what properties make it improper.

3.2.2 Mastery

- For functions $f(x, y) \geq 0$, be able to determine whether an improper double integral diverges or converges. Evaluate integrals which converge.

3.3 Triple Integrals, §14.5, §14.6

3.3.1 Passing

- Compute relatively simple triple integrals over bounded domains in cartesian coordinates.
- Know the transformations between cartesian coordinates and cylindrical and spherical coordinates as well as in the inverse transformations. Be able to compute the volume form for these coordinate systems, as well as for other given changes of variables.

- Can compute relatively simple triple integrals over bounded domains in cylindrical and spherical coordinates, and with a given variable substitution.

3.3.2 Mastery

- Evaluate more difficult triple integrals in cartesian, cylindrical, and spherical coordinates. Be able to introduce and use general variable transformations in order to solve a difficult triple integral.

4 Vector-Valued Functions of Multiple Variables

4.1 Curves in $\mathbb{R}^n$ and Parametrizations, §11.1 and §11.3

In which we studied vector valued functions of one variable (aka curves), derivatives of such functions, and parametrizations of such functions.

4.1.1 Passing

- Understand and compute the average velocity, the velocity and the acceleration vectors, as well as the speed, from a parametric curve. Describe and sketch the motion of the corresponding particle.
- Obtain expressions for parametrized curves of intersection in simple cases (e.g. Problem 6 from Section 11.3 of *Adams and Essex*).

4.1.2 Mastery

- Obtain expressions for parametrized curves of intersection in more complex cases (e.g. Problem 9 from Section 11.3 of *Adams and Essex*).

4.2 Vector Fields, §15.1

4.2.1 Passing

- Understand the concept of a vector field, and be able to give examples.
- Given an expression for a simple vector field be able to make a representative sketch.
- Understand and be able to give the definition of an integral curve (field lines) of a vector field.
- Given a simple vector field, can obtain the equations of the integral curves and sketch them.

4.3 Reduction of Second Order ODE

4.3.1 Passing

- Be able to reduce a second order ordinary differential equation (ODE) to a system of first order ODE. Sketch the vector field corresponding to the right hand side, and the integral curves. Interpret the behavior the system.

5 Integrals and Derivatives of Vector-Valued Functions of Multiple Variables

5.1 Conservative Fields, §15.2

5.1.1 Passing

- Understand the concept of a conservative field, and be able to give the definition.
- For a conservative vector field $\bar{F}$ with potential ϕ, explain the relationship between the level curves of ϕ and the field lines of $\bar{F}$.
- Know the necessary conditions for a vector field to be conservative, and use these to show that a given vector field is conservative or not.
- Be able to compute the potential of a conservative vector field.

5.1.2 Mastery

- Can state and prove the path-independence theorem for conservative vector fields (cf. Theorem 1 page 883 of *Adams and Essex*).

5.2 Line Integrals, §15.3

5.2.1 Passing

- Be able to parameterize simple curves including lines, quadratics, circles, ellipses, and helixes.
- Can compute the arclength of a curve segment in the plane or the room in simple examples such as those listed above.
- For simple curves like those listed above, be able to compute the line integral of a function over the curve.

5.3 Line Integrals of Vector Fields, §15.4

5.3.1 Passing

- Understand and be able to interpret in physical terms the line integral of a vector field.

- For simple vector fields and simple curves (such as those listed above), be able to compute the line integral of the vector field over the curve.

5.4 Div, Grad, Curl, §16.1, §16.2

5.4.1 Passing

- Can compute the divergence and curl of a given vector field.
- Understand the intuitive concept of the divergence and the curl of a vector field.
- Understand and give the definition of solenoidal and irrotational vector fields.
- Can apply Theorem 4 page 917 of *Adams and Essex* to show a vector field is conservative.

5.4.2 Mastery

- Can state and prove the vector field identities that say every conservative field is irrotational, and the curl of any vector field is solenoidal (cf. page 915-917 of *Adams and Essex*).

5.5 Green's Theorem in the Plane, §16.3

5.5.1 Passing

- Understand the idea of Green's theorem in the plane and can state the result.
- Can use Green's theorem to compute the area bounded by a closed curve.
- Can use Green's theorem to evaluate a line integral.

5.5.2 Mastery

- State and prove Green's theorem in the plane (Theorem 6 page 922 of *Adams and Essex*).

5.6 Surface Integrals, §15.5 and §15.6

5.6.1 Passing

- Understand the concepts of an oriented surface, unit normal vector, oriented boundary of a surface, smooth surface, and closed surface.
- Know the surface element for simple surfaces such as boxes, spheres, and cylinders. Be able to compute the integral of a function over these surfaces.
- Familiar with the concept of flux.
- Can compute the flux of a vector field through simple surfaces such as boxes, spheres, cylinders.

5.6.2 Mastery

- Can compute surface integrals and flux integrals for surfaces described by the graph of a function of two variables.

- Can compute surface integrals and flux integrals for surfaces with one-to-one projection into the xy-plane and described by equation of the form $G(x, y, z) = 0$, (for example page 901 of *Adams and Essex*).

5.7 Divergence Theorem, §16.4

5.7.1 Passing

- Know the idea and equation of the divergence theorem.

- Be able to apply the divergence theorem to compute a surface integral or a volume integral in simple situations.

5.7.2 Mastery

- Can use the divergence theorem to find a vector field in situations with symmetry. For example find the electric field of a spherically, cylindrically, or planar symmetric charge distribution.

- State and prove the divergence theorem (Theorem 8 page 925 of *Adams and Essex*).