MVE041 Flervariabelanalys 2015 Passing/Mastery Week 2

1 Passing Part

§12.4 Higher Order Derivatives

- Compute n-th order partial derivatives of a function.

§12.5 The Chain Rule

- Use appropriate versions of the chain rule to compute the partial derivatives of functions of the form

$$z = f(x, y), \quad \text{where } x = u(t), \text{ and } y = v(t),$$

and

$$z = f(x, y), \quad \text{where } x = u(t, s), \text{ and } y = v(t, s).$$

§12.6 Linear Approximations

- Be able to use the tangent plane at a point to approximate the value of a function at a nearby point.
- Use the differential of a function at a point to estimate the function value at a nearby point, and to approximate percentage errors.
- Compute the linearization of a vector-valued function of a vector variable at a point (i.e. using the Jacobian derivative).

§12.7 Gradients and Directional Derivatives

- Understand and be able to give the definition of the gradient of a function. Given a function be able to compute its gradient.
- Know and understand the geometric properties of the gradient vector.
- Be able to compute the directional derivative of a function in the direction of a specified vector and evaluate this at a point.
- For a real-valued function of two real variables use the gradient at a point to compute the equation for the tangent plane at that point, and the equation for a line tangent to level curve passing through that point.

- For a real-valued function of three real variables use the gradient at a point to compute the equation for the tangent plane at that point

§12.9 Taylor Series

- Be able to write down the Taylor series for a general real-valued function of n real variables.
- Explicitly compute the Taylor series for a specified real-valued function of two real variables using the Taylor formula as well as using known expressions for functions of one variable.

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2 Mastery Part

§12.4

- Show that a function is harmonic.

§12.6

- Give the definition of a differentiable function of two variables (Def. 5 of *Adams and Essex*).
- State the mean value theorem for a real-valued function of two real variables (Theorem 3 in 12.6 of *Adams and Essex*).
- Understand the relationship between continuity, existence of partial derivatives, and differentiability for a function. What property of the first partial derivatives of a function guarantees that the function is differentiable at a point (cf. Theorem 4 in 12.6 of *Adams and Essex*).

§12.7

- Be able to prove that the gradient vector of a function at a point is orthogonal to the level curve of the function at that point (cf. Theorem 6 of 12.7 of *Adams and Essex*).
- Be able to state the definition of the directional derivative. Prove that the directional derivative can be computed using the gradient (cf. Theorem 7 of 12.7 of *Adams and Essex*).

- Use the gradient to prove the equation of the tangent plane to the graph of a function of two variables at a point (cf. Example 7 pg 724 of *Adams and Essex*).

§12.8 Implicit Functions

- Understand the relationship between the Jacobian determinant and transformations of $\mathbb{R}^n$. Be able to state the implicit function theorem for one equation in three variables.
- Be able to compute specified partial derivatives of functions implicitly defined by equations of two or three variables. Also specify the domain on which these expressions are valid.