Lösningsförslag till dugga MVE045 den 25/9 2010

1) a) $$\frac{\tan 3x}{2x} = \underbrace{\frac{\sin 3x}{3x}}_{\to 1} \cdot \underbrace{\frac{1}{\cos 3x}}_{\to 1} \cdot \frac{3}{2} \to \frac{3}{2} \text{ då } x \to 0$$

Svar: $\lim_{x\to 0} \frac{\tan 3x}{2x} = \frac{3}{2}$

b) $$\frac{\tan 3x}{x^2} = \underbrace{\frac{\sin 3x}{3x}}_{\substack{\to 1\\ x\to 0}} \cdot \underbrace{\frac{1}{\cos 3x}}_{\substack{\to 1\\ x\to 0}} \cdot \underbrace{\frac{1}{x}}_{\substack{\to +\infty \text{ då } x\to 0+\\ \to -\infty \text{ då } x\to 0-}}$$

Alltså $\lim_{x\to 0+} \frac{\tan 3x}{x^2} = +\infty$, $\lim_{x\to 0-} \frac{\tan 3x}{x^2} = -\infty$

Svar: $\lim_{x\to 0} \frac{\tan 3x}{x^2}$ existerar ej

c) $$x^{\sin x} = e^{\sin x \cdot \ln x} = e^{\frac{\sin x}{x} \cdot x \ln x}$$

$\frac{\sin x}{x} \to 1$, $x \to 0+$, $x \ln x \to 0$, $x \to 0+$ och

e^t kontinuerlig i 0.

Alltså $\lim_{x\to 0+} x^{\sin x} = 1$

Svar: $\lim_{x\to 0+} x^{\sin x} = 1$

d) $$\frac{\arcsin x}{x} = \{ y = \arcsin x \iff \sin y = x \ \& \ y \in [-\tfrac{\pi}{2}, \tfrac{\pi}{2}] \} =$$

$$= \frac{y}{\sin y} = \{ x \to 0 \text{ om och endast om } y \to 0 \}$$

$$\to 1 \text{ då } x \to 0$$

Svar: $\lim_{x\to 0} \frac{\arcsin x}{x} = 1$.

2) a) $f(x) = \ln(2 + |x|^3) = \ln(2 - x^3)$ för $x < 0$

$$f'(x) = \frac{1}{2-x^3} \cdot (-3x^2) = -\frac{3x^2}{2-x^3}$$

$$f'(-1) = -\frac{3}{3} = -1$$

Svar: -1

b) $g(x) = \arctan(\arcsin(x))$

$$g'(x) = \frac{1}{1 + (\arcsin(x))^2} \cdot \frac{1}{\sqrt{1-x^2}}$$

$$g'(\tfrac{1}{2}) = \frac{1}{1 + (\frac{\pi}{6})^2} \cdot \frac{1}{\sqrt{\frac{3}{4}}} = \frac{2}{\sqrt{3}} \cdot \frac{36}{36 + \pi^2}$$

Svar: $\frac{2}{\sqrt{3}} \cdot \frac{36}{36+\pi^2}$

c) $h(x) = x|x|$

$$h'(0) = \lim_{h \to 0} \frac{h|h| - 0}{h} = \lim_{h \to 0} |h| = 0$$

Svar: 0

3) Rita grafen till $f(x) = \dfrac{2x}{\sqrt{1+x^2}} - \arctan x$

$D_f = \mathbb{R} = D_{f'}$

$$f'(x) = \frac{2\sqrt{1+x^2} - 2x\frac{x}{\sqrt{1+x^2}}}{1+x^2} - \frac{1}{1+x^2} =$$

$$= \frac{1}{1+x^2}\left[2\sqrt{1+x^2} - 2\,\frac{1+x^2-1}{\sqrt{1+x^2}} - 1\right] =$$

$$= \frac{1}{1+x^2}\left[\frac{2}{\sqrt{1+x^2}} - 1\right] = \frac{1}{(1+x^2)\sqrt{1+x^2}}\left(2 - \sqrt{1+x^2}\right)$$

Alltså: $f'(x) = 0 \iff 2 = \sqrt{1+x^2} \iff 3 = x^2 \iff x = \pm\sqrt{3}$

Teckenstudium

x		$-\sqrt{3}$		$\sqrt{3}$	
$f'(x)$	$-$	0	$+$	0	$-$
$f(x)$	$\searrow$	Lok. min	$\nearrow$	Lok max	$\searrow$

$$f(\sqrt{3}) = \frac{2\sqrt{3}}{2} - \arctan\sqrt{3} = \sqrt{3} - \frac{\pi}{3}$$

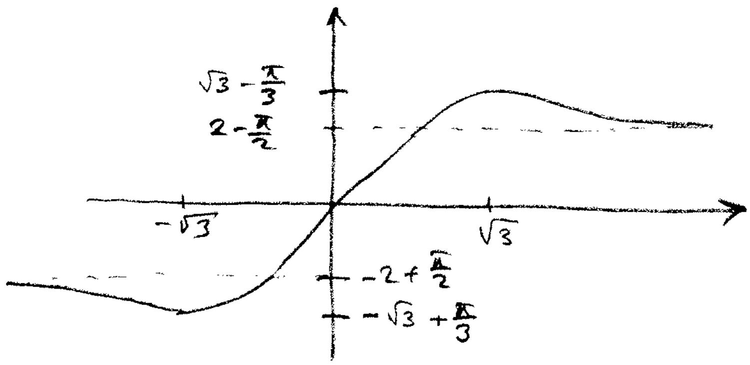
$$f(-\sqrt{3}) = -\frac{2\sqrt{3}}{2} - \arctan(-\sqrt{3}) = -\sqrt{3} + \frac{\pi}{3}$$

Asymptoter: vertikala asymptoter saknas

horisontella asymptoter

$$\lim_{x \to +\infty} f(x) = 2 - \frac{\pi}{2}$$

$$\lim_{x \to -\infty} f(x) = -2 + \frac{\pi}{2}$$



Obs: $f(x)$ udda funktion