

Formelblad för TMA043 och MVE085, 13/14

Trigonometri.

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin(x)\sin(y) = \frac{1}{2}(\cos(x-y) - \cos(x+y))$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\sin(x)\cos(y) = \frac{1}{2}(\sin(x-y) + \sin(x+y))$$

$$\cos(x)\cos(y) = \frac{1}{2}(\cos(x-y) + \cos(x+y))$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$$

Integralkatalog

$$\int x^a dx = \frac{x^{a+1}}{a+1} + C, \quad a \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \frac{1}{\cos^2 x} dx = \tan x + C$$

$$\int \frac{1}{\sin^2 x} dx = -\cot x + C$$

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C, \quad 0 < a \neq 1$$

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \arctan \frac{x}{a} + C, \quad a \neq 0$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

$$\int \frac{1}{\sqrt{a-x^2}} dx = \arcsin \frac{x}{\sqrt{a}} + C, \quad a > 0$$

$$\int \sqrt{a-x^2} dx = \frac{1}{2}x\sqrt{a-x^2} + \frac{a}{2} \arcsin \frac{x}{\sqrt{a}} + C, \quad a > 0$$

$$\int \frac{1}{\sqrt{x^2+a}} dx = \ln|x + \sqrt{x^2+a}| + C, \quad a \neq 0$$

$$\int \sqrt{x^2+a} dx = \frac{1}{2}(x\sqrt{x^2+a} + a \ln|x + \sqrt{x^2+a}|) + C$$

Maclaurinutvecklingar

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\sin x = \sum_{k=1}^{\infty} (-1)^{k-1} \frac{x^{2k-1}}{(2k-1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$(1+x)^\alpha = \sum_{k=0}^{\infty} \binom{\alpha}{k} x^k = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \dots, \quad |x| < 1, \quad \binom{\alpha}{k} = \frac{\alpha(\alpha-1)\dots(\alpha-k+1)}{k(k-1)\dots 1}$$

$$\ln(1+x) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, \quad -1 < x \leq 1$$

$$\arctan x = \sum_{k=1}^{\infty} (-1)^{k-1} \frac{x^{2k-1}}{2k-1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots, \quad |x| \leq 1$$

Övrigt

Masscentrum (x_T, y_T, z_T) för Ω ges av $x_T = \frac{\iiint_{\Omega} x \rho(x, y, z) dx dy dz}{\iiint_{\Omega} \rho(x, y, z) dx dy dz}$, analogt för y_T, z_T .

$\rho(x, y, z)$ är densiteten.