Hints for recommended exercises in Ref [2] Sec. 2.1 & Sec. 2.2

To solve the exercises, the following tools are required:

- Definition of binomial price of European derivatives, see Def. 3.3, Ref. [1]
- Recurrence formula for the binomial price of European derivatives, see Th. 3.1, Ref. [1]
- Hedging portfolio of standard European derivatives, Th. 3.2, Ref. [1]

1 Exercises Sec. 2.1

Exercise 1

We have a 1-period model, hence the portfolio is constant. The derivative is a call expiring at time $t = 1$ and with pay-off $Y = (S(1) - K)_+$ Letting $t = 1$ in (3.9b), Ref. [1], we obtain

$$h_S = \frac{1}{S(0)} \frac{\Pi_Y^u(1) - \Pi_Y^d(1)}{e^u - e^d}.$$

As $t = 1$ is the time of maturity, $\Pi_Y^u(1) = Y(u) = (S(0)e^u - K)_+$, $\Pi_Y^d(1) = Y(d) = (S(0)e^d - K)_+$. As $S(0)e^d < K < S(0)e^u$, then $Y(u) > 0$ and $Y(d) = 0$.

Exercise 2

Very similar to Exercise 1

Exercise 5

Using (3.7) in Ref. [1] with $N = 1$ we obtain

$$\Pi_Y(0) = e^{-r}[q_u Y(u) + q_d Y(d)] = e^{-r} q_u Y(u),$$

where we used that $Y(d) = 0$ (see ex. 1 above). As $q_u < 1$, then $\Pi_Y(0) < e^{-r} Y$.

2 Exercises Sec. 2.2

Exercise 2

This is a non-standard European derivative, hence to compute its value we use the definition of binomial price. Letting $t = 0$ and $N = 2$ in (3.3) Ref. [1], we obtain

$$\Pi_Y(0) = e^{-2r}(q_u^2 Y(u,u) + q_u q_d Y(u,d) + q_d q_u Y(d,u) + q_d^2 Y(d,d)))$$

Here $Y = ((S(0)S(1)S(2))^{1/3} - K)_+$ is the pay-off. Now compute the pay-off along any possible path, e.g.,

$$Y(u,u) = ((S(0)S(0)e^u S(0)e^{2u})^{1/3} - K)_+ = (S(0)e^u - K)_+$$

Some of the pay-offs will be zero because of the given inequality on the strike price and thus these paths do not contribute in the sum above.

Exercise 3

Follow the argument in Exercise 2

Exercise 4

The argument is the same as in exercises 2 and 3. For instance, for $T = 1$ we have

$$\Pi_Y(0) = e^{-r}[q_u Y(u) + q_d Y(d)] = e^{-r}[q_u(S(0)e^u - \min(S(0), S(0)e^u)) + q_d(S(0)e^d - \min(S(0), S(0)e^d))]$$

As $u > 0$ and $d < 0$, $\min(S(0), S(0)e^u) = S(0)$, $\min(S(0), S(0)e^d) = S(0)e^d$. Hence

$$\Pi_Y(0) = e^{-r}[q_u(S(0)e^u - S(0))] = S(0)e^{-r}[q_u(e^u - 1)].$$

Using $u = -d = 0.1$ and $r = 0.05$ (and a calculator...) we obtain $\Pi_Y(0) \approx 0.0731 S(0)$.

Exercise 6

The solution of this exercise is a bit more elaborated. It can be found in the 2013 old exam *pdf2* (see course homepage)