Lösningar till MVE 115, AT, 2014-01-14

① $f(1, \frac{\pi}{2}, \pi) = 1-\pi$, så nivåytan är
$\underline{x\sin y + 2y\cos z = 1-\pi}$.
grad $f = (\sin y,\ x\cos y + 2\cos z,\ -2y\sin z)$ och
$\nabla f(1, \frac{\pi}{2}, \pi) = (1, -2, 0)$, en normal till tangentplanet som då kan skrivas
$1(x-1) - 2(y-\frac{\pi}{2}) = 0$, dvs $\underline{x - 2y = 1-\pi}$

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② Parameterisering: $\begin{cases} x = \cos t \\ y = \sin t \end{cases}$ $\quad dx = -\sin t\, dt$, $\ dy = \cos t\, dt$ $\quad 0 \le t \le \frac{\pi}{2}$

$$I = \int_0^{\pi/2} [(\cos t + \sin t)(-\sin t) + \cos t \sin t \cos t]\, dt =$$
$$= \int_0^{\pi/2} (\sin t(\cos^2 t - \cos t) - \sin^2 t)\, dt =$$
$$= \left[-\frac{\cos^3 t}{3} + \frac{\cos^2 t}{2}\right]_0^{\pi/2} - \int_0^{\pi/2} \frac{1}{2} - \frac{1}{2}\cos 2t\, dt =$$
$$= \frac{1}{3} - \frac{1}{2} - \frac{\pi}{4} = \underline{\underline{-\frac{1}{6} - \frac{\pi}{4}}}$$

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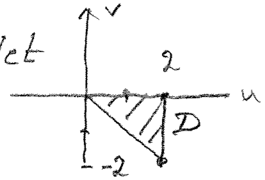
③

Variabelsubst: $\begin{cases} u = x+y \\ v = x-y \end{cases}$ ger området D: fig. $\left|\frac{d(x,y)}{d(u,v)}\right| = \frac{1}{2}$



$$I = \frac{1}{2}\int_0^2 \left(\int_{-u}^0 e^{-v/u}\, dv\right) du = \frac{1}{2}\int_0^2 -u\left[e^{-v/u}\right]_{-u}^0 du =$$
$$= \frac{1}{2}(e-1)\int_0^2 u\, du = \frac{1}{2}(e-1)\left[\frac{u^2}{2}\right]_0^2 = \underline{\underline{e-1}}$$

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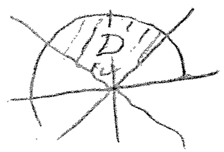
④ Greens formel ger



$$\oint_C (x+y^3)\,dx - (y+x^3)\,dy =$$
$$= \iint_D \frac{\partial}{\partial x}(-y-x^3) - \frac{\partial}{\partial y}(x+y^3)\; dx\,dy =$$
$$= \iint_D -3(x^2+y^2)\, dx\,dy = [\text{polära kordinater}] =$$
$$-3\int_{\pi/4}^{3\pi/4}\left(\int_0^1 r^2 \cdot r\,dr\right) d\theta = -\frac{3\pi}{2}\left[\frac{r^4}{4}\right]_0^1 = \underline{\underline{-\frac{3\pi}{8}}}$$

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⑤ $f'_x = \frac{y^2-1}{(xy-1)^2} = 0 \Leftrightarrow y = \pm 1$, $\quad f'_y = \frac{1-x^2}{(xy-1)^2} = 0 \Leftrightarrow x = \pm 1$

Stationära punkter: $(1,-1)$ & $(-1,1)$. (obs $xy \neq 1$)

$f''_{xx} = -2y(y^2-1)(xy-1)^{-3} = 0$; båda punkterna

$f''_{yy} = -2x(1-x^2)(xy-1)^{-3} = 0$ — " —

$f''_{xy} = \frac{2y(xy-1) - 2x(y^2-1)}{(xy-1)^3} = -\frac{1}{2}$ resp $\frac{1}{2}$ — " —

$\mathcal{H}(1,-1) = \begin{bmatrix} 0 & -1/2 \\ -1/2 & 0 \end{bmatrix}$ Eftersom $D = -\frac{1}{4} \neq 0$ men $D_1 = 0$

så är $\mathcal{H}$ indefinit. Samma gäller i $(-1,1)$

$\underline{(1,-1) \text{ och } (-1,1) \text{ är sadelpunkter}}$

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⑥ $\mathbf{r}'_u \times \mathbf{r}'_v = (2v, 2u, 0) \times (2u, 0, 4v) = 4(2uv, -2v^2, u^2)$

$$A = \int_1^2\left(\int_1^3 |\mathbf{r}'_u \times \mathbf{r}'_v|\, du\right) dv = \int_1^2\left(\int_1^3 4\sqrt{u^4 + 4v^4 + 4u^2v^2}\, du\right) dv =$$
$$= 4\int_1^2\left(\int_1^3 u^2 + 2v^2\, du\right) dv = 8\int_1^2 u^2\, du + 8\int_1^3 v^2\, dv =$$
$$= 8\left(\left(\frac{8}{3} - \frac{1}{3}\right) + \left(\frac{27}{3} - \frac{1}{3}\right)\right) = \underline{88}$$