Lösningar till MVE 115, 140308

① a) $\nabla f = (2xy + e^x y,\ x^2 + e^x) = (i\ (0,2)) = (2,1)$

$\mathbf{v} = \frac{(3,4)}{|(3,4)|} = \frac{1}{5}(3,4) \Rightarrow f'_{\mathbf{v}} = \frac{1}{5}(3,4)\cdot(2,1) = \underline{\underline{2}}$

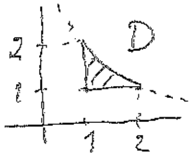
b) $0 = x^2 y + e^x y - z = F(x,y,z)$. $\quad f(0,2) = 2$.

$\nabla F = (2xy + e^x y,\ x^2 + e^x,\ -1) = (i\ (0,2,2)) = (2,1,-1)$.

Normalen: $\underline{\underline{\mathbf{r} = (0,2,2) + t(2,1,-1)}}$.

---

② $\int_1^2 \left(\int_1^{2/x} \frac{x}{y^2}\,dy\right)dx = \int_1^2 x\left[-\frac{1}{y}\right]_1^{2/x} dx =$



$= \int_1^2 x\left(-\frac{x}{2} + 1\right)dx = \int_1^2 \left(x - \frac{x^2}{2}\right)dx =$

$= \left[\frac{x^2}{2} - \frac{x^3}{6}\right]_1^2 = 2 - \frac{8}{6} - \left(\frac{1}{2} - \frac{1}{6}\right) = \underline{\underline{\frac{1}{3}}}$

---

③ a) $\left|\frac{x^2y^3}{(x^2+y^2)^2} - 0\right| = \left\{\begin{matrix}\text{polära}\\ \text{koordinater}\end{matrix}\right\} =$

$= \left|\frac{r^2\cos^2\theta\, r^3\sin^3\theta}{(r^2)^2}\right| = r|\cos^2\theta\sin^3\theta| \le r \to 0$

då $r \to 0$, oberoende av θ. Klart.

b) $f(x,y) = \frac{x^2y^2}{(x^2+y^2)^2} \Rightarrow f(x,0) = 0 \to 0$

då $(x,y) \to (0,0)$. Men längs linjen $y = x$

får vi $f(x,x) = \frac{x^4}{(2x^2)^2} = \frac{1}{4} \to \frac{1}{4}$.

Olika gränsvärden längs olika linjer

$\Rightarrow$ gränsvärde saknas

---

④ Minimera $(\text{avståndet})^2$, dvs $f(x,y) = x^2 + y^2$

under bivillkoret $g(x,y) = x^2y - 1 = 0$

Med $F(x,y,\lambda) = x^2 + y^2 + \lambda(x^2y - 1)$ får

vi att $\nabla F = 0 \Leftrightarrow \begin{cases} 1)\ 2x + 2xy\lambda = 0 \\ 2)\ 2y + x^2\lambda = 0 \\ 3)\ x^2y - 1 = 0 \end{cases}$

$x\cdot 1) - 2y\cdot 2)$ ger

$2x^2 - 4y^2 = 0$, dvs $x^2 = 2y^2$. Insatt i 3):

$2y^3 = 1$, $\quad y = 2^{-1/3}$ och $x = \sqrt{2}\,y = 2^{\frac{1}{2}-\frac{1}{3}} = 2^{1/6}$

punkten: $\underline{\underline{(2^{1/6},\ 2^{-1/3})}}$ Avståndet $= \underline{\underline{\sqrt{2^{1/3} + 2^{-2/3}} = 2^{-1/3}\sqrt{3}}}$

---

⑤ Parameterframställning: $\mathbf{r} = (\sqrt{2}\cos u, \sqrt{2}\sin u, v)$

$\mathbf{r}'_u \times \mathbf{r}'_v = (-\sqrt{2}\sin u, \sqrt{2}\cos u, 0) \times (0,0,1) = \sqrt{2}(\cos u, \sin u, 0)$

$F(\mathbf{r}(u,v)) = \left(\frac{1}{\sqrt{2}}\cos u, \frac{1}{\sqrt{2}}\sin u, \frac{v}{2}\right)$

Flödet $= \int_0^{2\pi}\left(\int_{-2}^{2}\left(\frac{1}{\sqrt{2}}\cos u, \frac{1}{\sqrt{2}}\sin u, \frac{v}{2}\right)\cdot(\cos u, \sin u, 0)\,dv\right)du$

$= \int_0^{2\pi}\left(\int_{-2}^{2} 1\,dv\right)du = \underline{\underline{8\pi}}$

---

⑥ $\iiint_D \frac{x^2}{(x^2+y^2+z^2)^a}\,dxdydz = \{\text{sfäriska koord.}\} =$

$= \int_0^{2\pi}\left(\int_0^{\pi}\left(\int_1^{\infty} \frac{r^2\cos^2\theta}{(r^2)^a}\, r^2\sin\theta\,dr\right)d\theta\right)d\varphi =$

$= 2\pi\int_0^{\pi}\cos^2\theta\sin\theta\left(\int_1^{\infty}\frac{1}{r^{2a-4}}\,dr\right)d\theta = \{\text{om } 2a-4>1\}$

$= \frac{2\pi}{3}\left[-\cos^3\theta\right]_0^{\pi}\left[\frac{r^{5-2a}}{5-2a}\right]_1^{\infty} = \underline{\underline{\frac{4\pi}{6a-15}}}$

$\underline{\underline{\text{konvergent då } a > 5/2}}$