On discounted costs

Primal problem

$$\begin{array}{ll} \max & \sum_{t=1}^{T} x_t\,, \\ \text{st} & x_t \le d_t, \quad t = 1, \dots, T, \\ & x_t \ge 0, \quad t = 1, \dots, T. \end{array}$$

$$x_t^* = d_t$$

discounted primal problem

$$\begin{array}{ll} \max & \sum_{t=1}^{T} \frac{1}{(1+r)^{t-1}} x_t\,, \\ \text{st} & x_t \le d_t, \quad t = 1, \dots, T, \\ & x_t \ge 0, \quad t = 1, \dots, T. \end{array}$$

$$x_t^* = d_t$$

Corresponding dual problem

$$\begin{array}{ll} \min & \sum_{t=1}^{T} d_t y_t\,, \\ \text{st} & y_t \ge 1, \quad t = 1, \dots, T, \\ & y_t \ge 0, \quad t = 1, \dots, T. \end{array}$$

$$y_t^* = 1$$

corresponding discounted dual problem

$$\begin{array}{ll} \min & \sum_{t=1}^{T} d_t y_t\,, \\ \text{st} & y_t \ge \frac{1}{(1+r)^{t-1}}\,, \quad t = 1, \dots, T, \\ & y_t \ge 0, \quad t = 1, \dots, T. \end{array}$$

$$y_t^* = \frac{1}{(1+r)^{t-1}}$$