# Binomiska ekvationer

Ex 1) Lös ekvationen

$z^6 = 1$.

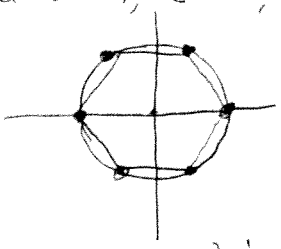
Skriv polär form: $z = re^{i\theta}$, $1 = e^{n\cdot 2\pi i}$

$r^6 e^{i6\theta} = e^{n\cdot 2\pi i}$

$r^6 = 1$, $6\theta = n\cdot 2\pi$

$r = 1$, $\theta = n\cdot \frac{2\pi}{6} = n\cdot \frac{\pi}{3}$

Lösningar: $1, e^{i\pi/3}, e^{i2\pi/3}, -1, e^{i4\pi/3}, e^{i5\pi/3}$



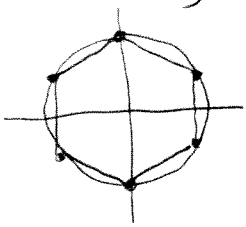
Eller, på kartesisk form:

$1, \frac{1}{2}+\frac{\sqrt{3}}{2}i, -\frac{1}{2}+\frac{\sqrt{3}}{2}i, -1, -\frac{1}{2}-\frac{\sqrt{3}}{2}i, \frac{1}{2}-\frac{\sqrt{3}}{2}i$

2) $z^6 = -1 = e^{(\pi+n\cdot 2\pi)i}$

$r^6 = 1$, $6\theta = \pi + n\cdot 2\pi$, $\theta = \frac{\pi}{6} + n\cdot \frac{\pi}{3}$

Lösningar: $e^{i\pi/6}, e^{i\pi/2}, e^{i5\pi/6}, e^{i3\pi/2}, e^{i11\pi/6}$



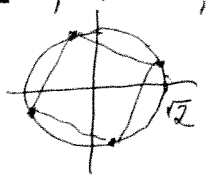
Eller:

$\frac{\sqrt{3}}{2}+\frac{1}{2}i, i, -\frac{\sqrt{3}}{2}+\frac{1}{2}i, -\frac{\sqrt{3}}{2}-\frac{1}{2}i, -i, \frac{\sqrt{3}}{2}-\frac{1}{2}i$

3) $z^4 = 4i = 4e^{i(\pi/2 + n\cdot 2\pi)}$

$r^4 = 4$, $r = \sqrt{2}$. $4\theta = \frac{\pi}{2} + n\cdot 2\pi$, $\theta = \frac{\pi}{8} + n\cdot \frac{\pi}{2}$

Lösningar $\sqrt{2}e^{\frac{\pi}{8}i}, \sqrt{2}e^{\frac{5\pi}{8}i}, \sqrt{2}e^{\frac{9\pi}{8}i}, \sqrt{2}e^{\frac{13\pi}{8}i}$




$\left(\frac{\sqrt{2+\sqrt{2}}}{2}+i\frac{\sqrt{2-\sqrt{2}}}{2}, -\frac{\sqrt{2-\sqrt{2}}}{2}+i\frac{\sqrt{2+\sqrt{2}}}{2}\right.$,

$\left.-\frac{\sqrt{2+\sqrt{2}}}{2}-i\frac{\sqrt{2-\sqrt{2}}}{2}, \frac{\sqrt{2-\sqrt{2}}}{2}-i\frac{\sqrt{2+\sqrt{2}}}{2}\right)$

Övning: Lös ekvationerna a) $z^5 = 4\sqrt{2}$ b) $z^3 = -8$

c) $z^3 = -2+2i$ Rita figurer.

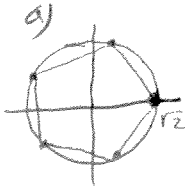
c) $z^3 = -(1+i)$

Svar: a) $\sqrt{2}, \sqrt{2}e^{2\pi i/5}, \sqrt{2}e^{4\pi i/5}, \sqrt{2}e^{6\pi i/5}, \sqrt{2}e^{8\pi i/5}$

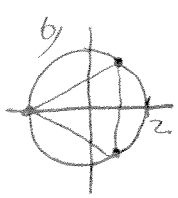
b) $2e^{i\pi/3}, -2, 2e^{-i\pi/3}$

c) $\sqrt{2}e^{i\pi/4}, \sqrt{2}e^{11\pi i/12}, \sqrt{2}e^{19\pi i/12}$

a)




b)




c)