(Lösningar, MVE 335, 09 10 21.)

1

$2x+x-1=0 \iff x+\frac{1}{2}x-\frac{1}{2}=0 \iff$

a) $x=-\frac{1}{4}\pm\sqrt{(-\frac{1}{4})^2+\frac{1}{2}} = -\frac{1}{4}\pm\sqrt{\frac{9}{16}} = -\frac{1}{4}\pm\frac{3}{4}$

$x_1=-\frac{1}{4}+\frac{3}{4}=\underline{\underline{\frac{1}{2}}}$, $\quad x_2=-\frac{1}{4}-\frac{3}{4}=\underline{\underline{-1}}$

b) $2x+x-1=2(x+\frac{1}{2}x-\frac{1}{2})=\underline{2(x-\frac{1}{2})(x+1)}=\underline{\underline{(2x-1)(x+1)}}$.

c) $p(x)=2(x+\frac{1}{2}-\frac{1}{2})=\{\text{kvadratkomplettera}\}=$

$=2\left[(x+\frac{1}{4})^2-(\frac{1}{4})^2-\frac{1}{2}\right]=2\left[(x+\frac{1}{4})^2-\frac{9}{16}\right]=$

$=2(x+\frac{1}{4})^2-\frac{9}{8}$. Minsta värde $=\underline{\underline{-\frac{9}{8}}}$

(då $x=-\frac{1}{4}$)

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2 $a+b=(-3,1)+(1,2)=(-2,3)$

a) $|a+b|=\sqrt{(-2)^2+3^2}=\underline{\underline{\sqrt{13}}}$

b) Skalärprodukt: $a\cdot b=|a||b|\cos\theta$.

I koordinater: $-3\cdot1+1\cdot2=\sqrt{(-3)^2+1^2}\sqrt{1^2+2^2}\cos\theta$

$-1=\sqrt{10}\sqrt{5}\cos\theta$, $\quad \theta=\underline{\underline{\arccos(-1/\sqrt{50})\approx 98^\circ}}$

3

a) $9\,\dfrac{3^{-3}x^{-6}y^{15}\cdot 2^{-5}}{2^{-5}x^{-5}y^{6}y^{8}}=9\cdot3^{-3}x^{-6}x^{5}y^{15}y^{-14}2^{-5}2^{5}=\underline{\underline{\frac{y}{3x}}}$

b) $\dfrac{x^2(4x^2-1)}{x^2(2x+1)}=\dfrac{(2x-1)(2x+1)}{2x+1}=\underline{\underline{2x-1}}$

4

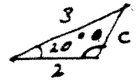
a) $|w|=\sqrt{2^2+(-1)^2}=\sqrt{4+1}=\underline{\underline{\sqrt{5}}}$

$\overline{w}=2+i$

b) $\dfrac{11+7i}{2-i}=\dfrac{(11+7i)(2+i)}{(2-i)(2+i)}=\dfrac{22-7+14i+11i}{4+1-2i+2i}=$

$=\dfrac{15+25i}{5}=\underline{\underline{3+5i}}$

5 cosinussatsen ger



a) $c^2=2^2+3^2-2\cdot2\cdot3\cos\theta=$

$=13-12\cos20^\circ$

$c=\sqrt{13-12\cos20^\circ}\approx\underline{1,31}$

Sinussatsen: $\dfrac{\sin\theta}{3}=\dfrac{\sin20^\circ}{1,31}$

$\theta=(\text{Trubbig vinkel})=180-\arcsin(\frac{3}{1,31}\sin20^\circ)\approx$

$\approx(180-52)^\circ=\underline{\underline{128^\circ}}$

3:e vinkeln $\approx180^\circ-20^\circ-128^\circ=\underline{\underline{32^\circ}}$

b) Arean =

$=\frac{1}{2}\cdot2\cdot3\sin20^\circ\approx1.03$

6

a) $\lg(x-1)(x+1)-\lg(x+1)=\lg(2x-3)$

$\iff \lg\dfrac{(x-1)(x+1)}{(x+1)}=\lg(2x-3)\iff\lg(x-1)=\lg(2x-3)$

$\iff x-1=2x-3\iff \underline{\underline{x=2}}$

b) $2^x+2^x\cdot2^{-1}=6\iff 2^x+\frac{1}{2}2^x=6\iff$

$\frac{3}{2}\cdot2^x=6\iff 2^x=4\iff \underline{\underline{x=2}}$

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7 $\sin2x=\pm\sqrt{\frac{3}{4}}=\pm\frac{\sqrt3}{2}$

1) $\sin2x=\frac{\sqrt3}{2}$

a) $2x=\frac{\pi}{3}+n\cdot2\pi$

$\underline{x=\frac{\pi}{6}+n\pi}$

b) $2x=\pi-\frac{\pi}{3}+n2\pi$

$\underline{x=\frac{\pi}{3}+n\pi}$

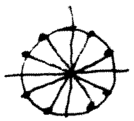
2) $\sin2x=-\frac{\sqrt3}{2}$

a) $2x=-\frac{\pi}{3}+n\cdot2\pi$

$\underline{x=-\frac{\pi}{6}+n\pi}$

b) $2x=\pi-(-\frac{\pi}{3})+n\cdot2\pi$

$\underline{x=\frac{2\pi}{3}+n\cdot\pi}$



8 Skriv polärt:

$-2+2i=2\sqrt2(-\frac{1}{\sqrt2}+\frac{1}{\sqrt2}i)=2\sqrt2(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4})=$

$=2\sqrt2\,e^{i\frac{3\pi}{4}}$. Med $z=re^{i\theta}$ får vi

$r^3e^{3i\theta}=2\sqrt2\,e^{\frac{3\pi}{4}i}$, dvs $\underline{r=\sqrt2}$ och

$3\theta=\frac{3\pi}{4}+n\cdot2\pi$ och $\theta=\frac{\pi}{4}+n\cdot\frac{2\pi}{3}$, $n=0,1,2$ ger

$\theta_1=\frac{\pi}{4}$, $\quad\theta_2=\frac{\pi}{4}+\frac{2\pi}{3}=\frac{11\pi}{12}$, $\quad\theta_3=\frac{\pi}{4}+\frac{4\pi}{3}=\frac{19\pi}{12}$

$z_1=\sqrt2\,e^{\frac{\pi}{4}i}$, $\quad z_2=\sqrt2\,e^{\frac{11\pi}{12}i}$

$z_3=\sqrt2\,e^{\frac{19\pi}{12}i}$