Lösningar till Matematik 1 för SI1, MVE335. 11 10 21

(1) a) $x^2 + 1^2 = 2^2$ ger $x^2 = 4 - 1 = 3$ och $\underline{x = \sqrt{3}}$


b) Arean $T = \sqrt{3} \cdot 1/2 = \underline{\frac{\sqrt{3}}{2}}$

c) $\sin\alpha = \frac{1}{2}$ ger $\underline{\alpha = 30^\circ}$ och $\underline{\beta = 60^\circ}$

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(2) a) $x = -\frac{4}{2} \pm \sqrt{\left(\frac{4}{2}\right)^2 - 3} = -2 \pm \sqrt{4-3} = -2 \pm 1$

$x_1 = \underline{-1}$, $x_2 = \underline{-3}$

b) $x = -2 \pm \sqrt{2^2 - 5} = -2 \pm \sqrt{-1} = -2 \pm i$

$z_1 = \underline{-2+i}$, $z_2 = \underline{-2-i}$

c) $z^2 + 4z + 5 = (z+2)^2 + 1 \geq 1$ om $z$ är reellt. $\underline{\text{Minsta värdet är } 1}$

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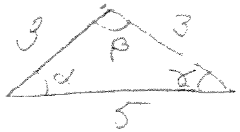
(3) Cosinussatsen:

$5^2 = 3^2 + 3^2 - 2 \cdot 3 \cdot 3 \cos\beta$,

$18\cos\beta = 18 - 25 = -7$

$\beta = \arccos\left(-\frac{7}{18}\right) \approx 113{,}8$




$\alpha + \gamma = 2\alpha = 180 - 113{,}8 = 67{,}2$, $\underline{\alpha = \gamma = 33{,}6}$

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(4) $|a| = \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$

a) $b = \frac{1}{5} a = \underline{\left(-\frac{3}{5}, \frac{4}{5}\right)}$ har belopp 1

b) $b \cdot a = 0$ om $b = (4, 3)$, $|b| = 5$ och $c = \frac{1}{5} b = \underline{\left(\frac{4}{5}, \frac{3}{5}\right)}$ är den sökta vektorn

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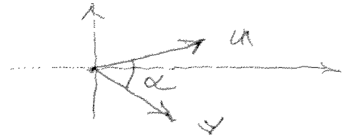
(5) $v = 2a - 3b = (2, 1) - (-1, 3) = (3, -2)$

$u = 2a + 3b = (2, 1) + (-1, 3) = (1, 4)$

$v \cdot u = (3, -2) \cdot (1, 4) = 3 - 8 = -5$




$|v| = \sqrt{3^2 + 2^2} = \sqrt{13}$

$|u| = \sqrt{1^2 + 4^2} = \sqrt{17}$

$u \cdot v = |u|\,|v| \cos\alpha$ ger $\underline{\cos\alpha = -\frac{5}{\sqrt{13}\sqrt{17}}}$

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(6) $z\bar{w} = (-2+3i)(1-i) = -2 - 3i^2 + 2i + 3i = 1 + 5i$

a) $\bar{z}w = \overline{z\bar{w}} = 1 - 5i$, $\underline{z\bar{w} + \bar{z}w = 2}$,

b) $|1+i| = \sqrt{1^2 + 1^2} = \sqrt{2}$,

$1 + i = \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i\right) = \sqrt{2}\left(\cos\frac{\pi}{4} + i \sin\frac{\pi}{4}\right) =$

$= \underline{\sqrt{2}\, e^{i\pi/4}}$

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(7) a) $\ln x = \ln \frac{2}{x+1}$, $x = \frac{2}{x+1}$, $x(x+1) = 2$

$x^2 + x - 2 = 0$, $x = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{8}{4}} = -\frac{1}{2} \pm \frac{3}{2}$

$\underline{x_1 = 1}$, $(x_2 = -2)$. $x = -2$ är falsk ty $\ln x$ finns ej om $x \leq 0$. Svar $\underline{x = 1}$.

b) $2^{x^2} = 2^3 \cdot 2^x$, $2^{x^2} = 2^{3+x}$ dvs

$x^2 = 3 + x$ och $x^2 - x - 2 = 0$

$x = \frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{8}{4}} = \frac{1}{2} \pm \frac{3}{2}$

$\underline{x_1 = 2}$, $\underline{x_2 = -1}$

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Skriv polärt $(re^{i\theta})^8 = 16\left(-\frac{1}{2} - \frac{\sqrt{3}}{2} i\right) = 16\, e^{i\frac{4\pi}{3}}$

$r^8 = 16$ ger $r = \sqrt{2}$ $8\theta = \frac{4\pi}{3} + n \cdot 2\pi$ ger

$\theta = \frac{\pi}{6} + n \cdot \frac{\pi}{4}$. En rot $(n=0)$:

$z = \sqrt{2}\, e^{i\frac{\pi}{6}} = \sqrt{2}\left(\cos\frac{\pi}{6} + i \sin\frac{\pi}{6}\right)$

$= \sqrt{2}\left(\frac{\sqrt{3}}{2} + i\,\frac{1}{2}\right) = \underline{\frac{\sqrt{6}}{2} + i\,\frac{\sqrt{2}}{2}}$